

Emergent

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Scenarios

Criteria

String Gas
Cosmology

Matrix Theory

Matrix Theory
Cosmology

Conclusions

Emergent Metric Space-Time from Matrix Theory

Robert Brandenberger
Physics Department, McGill University

Quantum Gravity and Cosmology Zoom Meeting, April 17 - 21
2023

Work in collaboration with S. Brahma and S. Laliberte
[arXiv:2106.11512](https://arxiv.org/abs/2106.11512), [arXiv:2206.12468](https://arxiv.org/abs/2206.12468)

Motivation

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Conclusions

- **Inflationary Scenario** is the **current paradigm** of **early Universe cosmology**.
- Inflation is usually analyzed using an **effective field theory** (EFT) framework.
- **Fundamental conceptual problems** for an EFT description of a rapidly expanding universe.
- **Unitarity problem, inconsistency with the 2nd law of thermodynamics.**
- We need to look beyond an EFT description of the early universe!
- **Matrix Theory Cosmology**: Emergent metric space-time and early universe from the **BFSS** matrix model.

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Outline

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Trans-Planckian Problem

J. Martin and R.B., *Phys. Rev. D*63, 123501 (2002)

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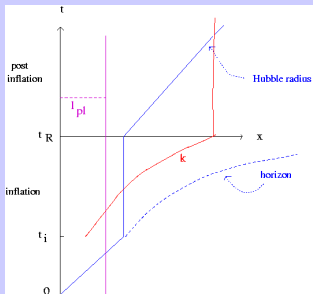
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Conclusions



- **Success of inflation:** At early times scales are inside the Hubble radius \rightarrow causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation.
- \rightarrow breakdown of effective field theory; new physics MUST be taken into account when computing observables from inflation.

Trans-Planckian Censorship Conjecture (TCC)

A. Bedroya and C. Vafa., arXiv:1909.11063

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Conclusions

No trans-Planckian modes exit the Hubble horizon.

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

$$H(t) \equiv \frac{\dot{a}}{a}(t)$$

$$\boxed{\frac{a(t_R)}{a(t_i)}|_{pl} < H(t_R)^{-1}}$$

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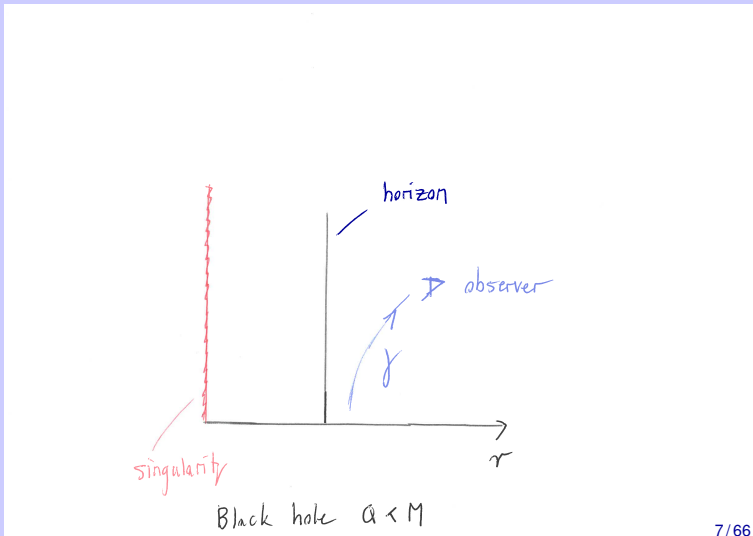
$$H(t) \equiv \frac{\dot{a}}{a}(t)$$

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Justification

R.B. arXiv:1911.06056

Analogy with Penrose's Cosmic Censorship Hypothesis:



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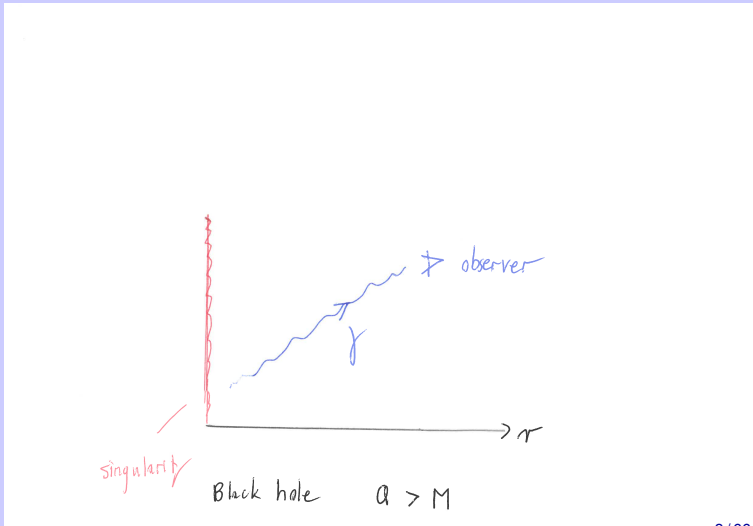
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Conclusions

- Effective field theory of General Relativity allows for solutions with **timelike singularities**: super-extremal black holes.
- → Cauchy problem not well defined for observer external to black holes.
- Evolution **non-unitary** for external observer.
- Conjecture: ultraviolet physics → **external observer** shielded from the **singularity** and **non-unitarity** by **horizon**.

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Cosmological Version of the Censorship Conjecture

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Translation

- Position space \rightarrow momentum space.
- Singularity \rightarrow trans-Planckian modes.
- Black Hole horizon \rightarrow Hubble horizon.

Observer measuring super-Hubble horizon modes must be shielded from trans-Planckian modes.

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Why Hubble Horizon?

R.B. arXiv:1911.06056; A. Bedroya and C. Vafa., arXiv:1909.11063

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Conclusions

- Recall: Fluctuations only oscillate on sub-Hubble scales.
- Recall: Fluctuations freeze out, become **squeezed states** and **classicalize** on super-Hubble scales.
- Demand: classical region be insensitive to trans-Planckian region.
- → no trans-Planckian modes ever exit Hubble horizon.

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Unitarity Problem

R.B. arXiv:1911.06056; A. Bedroya and C. Vafa., arXiv:1909.11063

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Conclusions

- Recall: **non-unitarity** of **effective field theory** in an expanding universe (N. Weiss, Phys. Rev. D32, 3228 (1985); J. Cotler and A. Strominger, arXiv:2201.11658).
- \mathcal{H} is the product Hilbert space of a harmonic oscillator Hilbert space for all **comoving** wave numbers k
- **UV cutoff: time dependent** $k_{max} : k_{max}(t)a(t)^{-1} = m_{pl}$
- Continuous mode creation \rightarrow **non-unitarity**.
- **Demand: classical region be insensitive to non-unitarity.**
- \rightarrow no trans-Planckian modes ever exit Hubble horizon.

Unitarity Problem

R.B. arXiv:1911.06056; A. Bedroya and C. Vafa., arXiv:1909.11063

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Effective Field Theory (EFT) and the CC Problem

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Conclusions

- EFT: expand **fields** in comoving Fourier space.
- Quantize each Fourier mode like a harmonic oscillator
→ ground state energy.
- Add up ground state energies → CC problem.
- The usual quantum view of the CC problem is an artefact of an EFT analysis!

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Application of the Second Law of Thermodynamics

S. Brahma, O. Alaryani and RB, arXiv:2005.09688

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Conclusions

- Consider **entanglement entropy density** $s_E(t)$ between sub- and super-Hubble modes.
- Consider an **phase of inflationary expansion**.
- $s_E(t)$ increases in time since the phase space of super-Hubble modes grows.
- **Demand:** $s_E(t)$ remain smaller than the post-inflationary thermal entropy.
- \rightarrow Duration of inflation is bounded from above, consistent with the TCC.

Application to EFT Description of Inflation

A. Bedroya, R.B., M. Loverde and C. Vafa., arXiv:1909.11106

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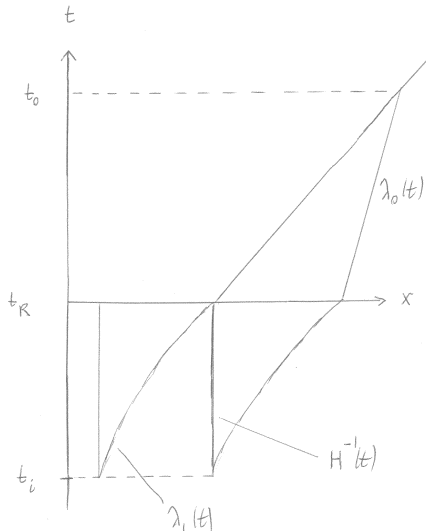
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Application to EFT Descriptions of Inflation

A. Bedroya, R.B., M. Loverde and C. Vafa., arXiv:1909.11106

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Conclusions

TCC implies:

$$\frac{a(t_R)}{a(t_*)} l_{pl} < H(t_R)^{-1}$$

Demanding that inflation yields a causal mechanism for generating CMB anisotropies implies:

$$H_0^{-1} \frac{a(t_0)}{a(t_R)} \frac{a(t_R)}{a(t_*)} < H^{-1}(t_*)$$

Implications

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Conclusions

Upper bound on the **energy scale of inflation**:

$$V^{1/4} < 3 \times 10^9 \text{GeV}$$

→ **upper bound** on the **primordial tensor to scalar ratio** r :

$$r < 10^{-30}$$

Note: Secondary tensors will be larger than the primary ones.

Implications for Dark Energy

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Conclusions

- Dark Energy cannot be a bare cosmological constant.
- Quintessence models of Dark Energy are constrained (L. Heisenberg et al. arXiv:2003.13283]

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Angular Power Spectrum of CMB Anisotropies

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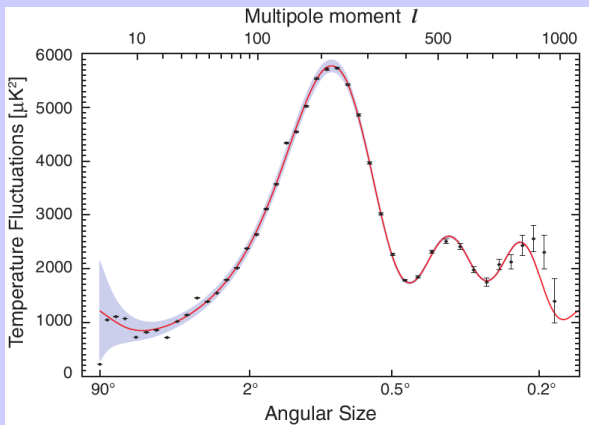
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Credit: NASA/WMAP Science Team

Early Work

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1970Ap&SS...7....3S

SMALL-SCALE FLUCTUATIONS OF RELIC RADIATION

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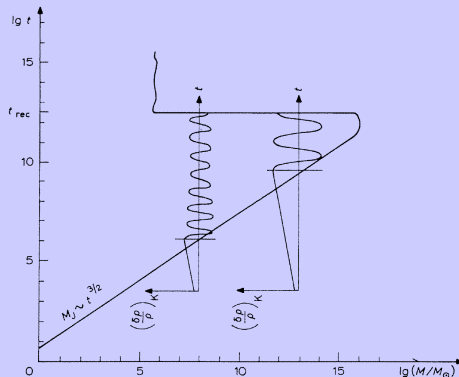


Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

1970 p

Predictions from 1970

R. Sunyaev and Y. Zel'dovich, *Astrophys. and Space Science* **7**, 3 (1970); P. Peebles and J. Yu, *Ap. J.* **162**, 815 (1970).

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Conclusions

- Given a **scale-invariant power spectrum of adiabatic fluctuations** on "super-horizon" scales before t_{eq} , i.e. standing waves.
- → "correct" power spectrum of galaxies.
- → **acoustic oscillations in CMB angular power spectrum.**
- → **baryon acoustic oscillations in matter power spectrum.**

Criteria for a Successful Early Universe Scenario

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Conclusions

- **Horizon \gg Hubble radius** in order for the scenario to solve the “horizon problem” of Standard Big Bang Cosmology.
- Scales of cosmological interest today **originate inside the Hubble radius at early times** in order for a causal generation mechanism of fluctuations to be possible.
- Mechanism for producing a **scale-invariant spectrum of curvature fluctuations** on super-Hubble scales.

Inflation as a Solution

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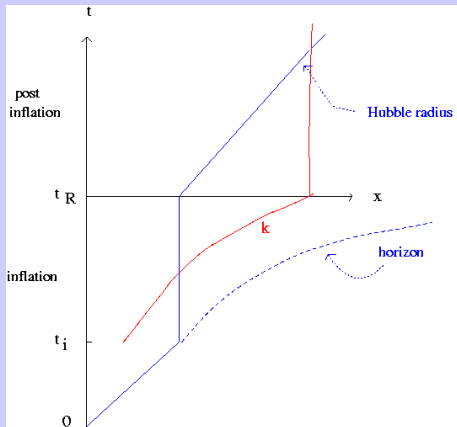
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Bouncing Cosmology as a Solution

F. Finelli and R.B., *Phys. Rev. D* **65**, 103522 (2002), D. Wands, *Phys. Rev. D* **60** (1999)

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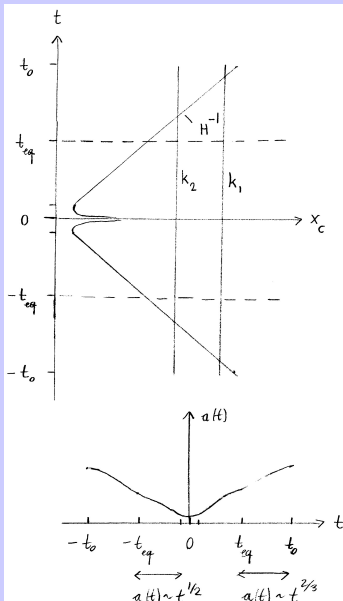
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Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B* 316:391 (1989)

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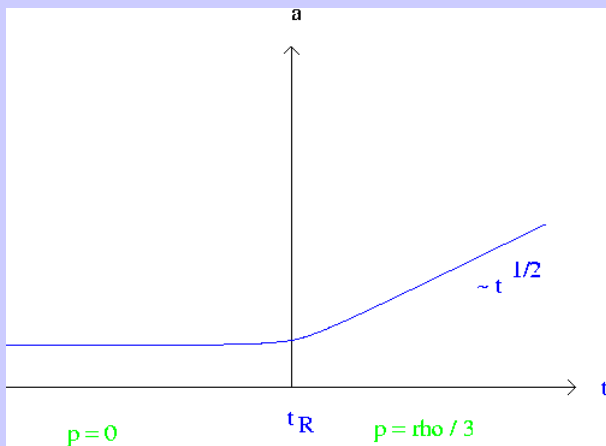
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Emergent Universe as a Solution

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett.* 97:021302 (2006)

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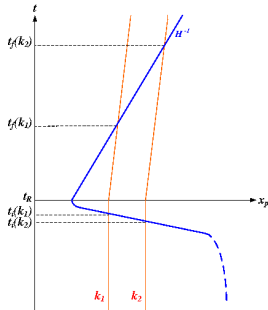
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Trans-Planckian Censorship and Cosmological Scenarios

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- **Bouncing cosmologies** are **consistent** with the TCC provided that the energy scale at the bounce is lower than the Planck scale.
- **Emergent cosmologies** are **consistent** with the TCC provided that the energy scale of the emergence phase is lower than the Planck scale.
- **Inflationary cosmologies** are **inconsistent** with the TCC unless the energy scale of inflation is fine tuned.

All early universe scenarios require going beyond EFT.

Trans-Planckian Censorship and Cosmological Scenarios

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Obtaining an Emergent Cosmology: String Gas Cosmology

R.B. and C. Vafa, *Nucl. Phys. B*316:391 (1989)

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Conclusions

Idea: make use of the **new symmetries** and **new degrees of freedom** which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- **New degrees of freedom**: string oscillatory modes
- Leads to a **maximal temperature** for a gas of strings, the Hagedorn temperature
- **New degrees of freedom**: string winding modes
- Leads to a **new symmetry**: physics at large R is equivalent to physics at small R

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T-Duality

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T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R$ $(n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level \rightarrow existence of D-branes

Adiabatic Considerations

R.B. and C. Vafa, *Nucl. Phys. B*316:391 (1989)

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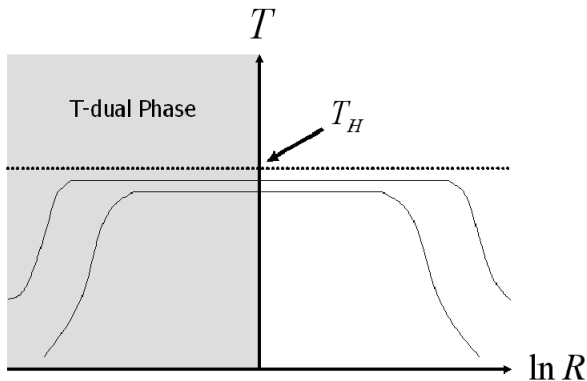
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Temperature-size relation in string gas cosmology



Background for string gas cosmology

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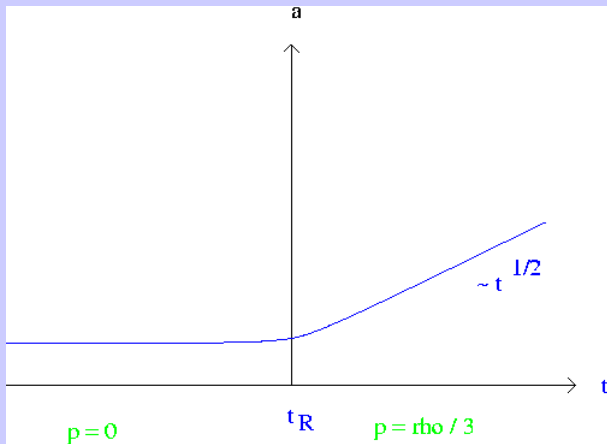
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Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett.* 97:021302 (2006)

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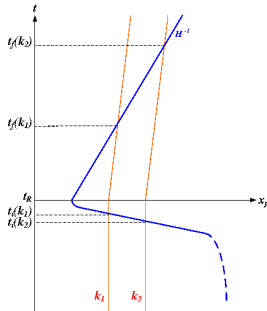
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N.B. Perturbations originate as thermal string gas fluctuations.

Method

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Conclusions

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k , convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Note: the matter correlation functions are given by partial derivatives of the **finite temperature string gas partition function** with respect to T (density fluctuations) or R (pressure perturbations).

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Extracting the Metric Fluctuations

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Conclusions

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta) \left((1 + 2\phi) d\eta^2 - [(1 - 2\phi)\delta_{ij} + h_{ij}] dx^i dx^j \right).$$

Inserting into the perturbed Einstein equations yields

$$\langle |\phi(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0_0(k) \delta T^0_0(k) \rangle,$$

$$\langle |h(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i_j(k) \delta T^i_j(k) \rangle.$$

Power Spectrum of Cosmological Perturbations

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Key ingredient: For **thermal** fluctuations:

$$\langle \delta \rho^2 \rangle = \frac{T^2}{R^6} C_V.$$

Key ingredient: For **string thermodynamics** in a compact space

$$C_V \approx 2 \frac{R^2 / \ell_s^3}{T (1 - T/T_H)}.$$

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Power spectrum of cosmological fluctuations

$$\begin{aligned}
 P_{\Phi}(k) &= 8G^2 k^{-1} \langle |\delta\rho(k)|^2 \rangle \\
 &= 8G^2 k^2 \langle (\delta M)^2 \rangle_R \\
 &= 8G^2 k^{-4} \langle (\delta\rho)^2 \rangle_R \\
 &= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H}
 \end{aligned}$$

Key features:

- **scale-invariant** like for inflation
- **slight red tilt** like for inflation

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Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett.* (2007)

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$$\begin{aligned}P_h(k) &= 16\pi^2 G^2 k^{-1} \langle |T_{ij}(k)|^2 \rangle \\&= 16\pi^2 G^2 k^{-4} \langle |T_{ij}(R)|^2 \rangle \\&\sim 16\pi^2 G^2 \frac{T}{\ell_s^3} (1 - T/T_H)\end{aligned}$$

Key ingredient for **string thermodynamics**

$$\langle |T_{ij}(R)|^2 \rangle \sim \frac{T}{\ell_s^3 R^4} (1 - T/T_H)$$

Key features:

- scale-invariant (like for inflation)
- slight blue tilt (unlike for inflation)

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Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

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Conclusions

Starting point: BFSS matrix model at high temperatures.

- BFSS model is a quantum mechanical model of 10 $N \times N$ Hermitean matrices.
- Note: no space!
- Note: no singularities!
- Note: BFSS matrix model is a proposed non-perturbative definition of M-theory: 10 dimensional superstring theory emerges in the $N \rightarrow \infty$ limit.

BFSS Model (bosonic sector)

T. Banks, W. Fischler, S. Shenker and L. Susskind, Phys. Rev. D **55**, 5112 (1997)

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Conclusions

$$L = \frac{1}{2g^2} [\text{Tr}(\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2)]$$

- $X_i, i = 1, \dots, 9$ are $N \times N$ Hermitean matrices.
- D_t : gauge covariant derivative (contains a matrix A_0)

't Hooft limit: $N \rightarrow \infty$ with $\lambda \equiv g^2 N = g_s l_s^{-3} N$ fixed.

Thermal Initial State

N. Kawahara, J. Nishimura and S. Takeuchi, JHEP **12**, 103 (2007)

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Conclusions

- Consider a high temperature state.
- At high temperatures, the bosonic sector of the (Euclidean) BFSS model is well approximated by the bosonic sector of the (Euclidean) **IKKT matrix model**.
- $S_{BFSS} = S_{IKKT} + \mathcal{O}(1/T)$
- Matsubara expansion:

$$X_i(t) = \sum_n X_i^n e^{2\pi i T t}$$

$$A_i \equiv T^{-1/4} X_i^0$$

IKKT Matrix Model

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B **498**, 467 (1997).

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Conclusions

Proposed as a non-perturbative definition of the IIB Superstring theory.

Action:

$$S_{IKKT} = -\frac{1}{g^2} \text{Tr} \left(\frac{1}{4} [A^a, A^b] [A_a, A_b] + \frac{i}{2} \bar{\psi}_\alpha (C \Gamma^a)_{\alpha\beta} [A_a, \psi_\beta] \right),$$

Partition function:

$$Z = \int dA d\psi e^{iS}$$

Emergent Time from Matrix Theory

Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1506.04795

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Conclusions

- Eigenvalues of A_0 become **emergent time**.
- Work in the basis in which A_0 is diagonal.
- Numerical studies: $\frac{1}{N} \langle \text{Tr} A_0^2 \rangle \sim \kappa N$
- $\rightarrow t_{max} \sim \sqrt{N}$
- $\rightarrow \Delta t \sim \frac{1}{\sqrt{N}}$
- \rightarrow infinite continuous time.

Note: $\sum_{n=0}^N n^2 = \frac{1}{6} N(N+1)(2N+1)$

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Emergent Space from Matrix Theory

Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1506.04795

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- Eigenvalues of A_0 become **emergent time**, continuous in $N \rightarrow \infty$ limit.
- Work in the basis in which A_0 is diagonal: A_i matrices elements decay when going away from the diagonal.
- $\sum_i \langle |A_i|_{ab}^2 \rangle$ decays when $|a - b| > n_c$
- $\sum_i \langle |A_i|_{ab}^2 \rangle \sim \text{constant}$ when $|a - b| < n_c$
- $n_c \sim \sqrt{N}$

Emergent Space from Matrix Theory

S. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540

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- Eigenvalues of A_0 become **emergent time**, continuous in $N \rightarrow \infty$ limit.
- Work in the basis in which A_0 is diagonal: A_i matrices elements decay when going away from the diagonal.
- Pick $n \times n$ blocks $\tilde{A}_i(t)$ about the diagonal ($n < n_c$)

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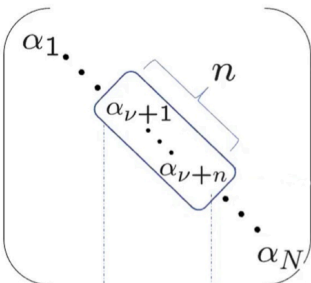
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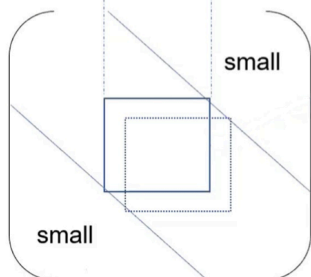
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Conclusions

$$A_0 = \left(\begin{array}{c} \alpha_1 \dots \alpha_{\nu+1} \dots \alpha_{\nu+n} \dots \alpha_N \\ \vdots \\ \vdots \end{array} \right)$$


The diagram shows a large matrix A_0 with elements $\alpha_1, \dots, \alpha_{\nu+1}, \dots, \alpha_{\nu+n}, \dots, \alpha_N$. A blue box highlights a sub-block of size n starting from $\alpha_{\nu+1}$. Dashed lines extend from the corners of this box to the corresponding corners of a smaller box in the matrix A_i below.

$$A_i = \left(\begin{array}{c} \text{small} \\ \vdots \\ \text{small} \end{array} \right)$$


The diagram shows a matrix A_i with a diagonal line. A blue box highlights a sub-block, labeled "small" in two places, indicating its size relative to the matrix.

Spontaneous Symmetry Breaking in Matrix Theory

J. Nishimura, PoS CORFU **2019**, 178 (2020) [arXiv:2006.00768 [hep-lat]].

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Conclusions

- Eigenvalues of A_0 become emergent time, continuous in $N \rightarrow \infty$ limit.
- Work in the basis in which A_0 is diagonal.
- Work in the basis in which A_0 is diagonal: A_i matrices become block diagonal.
- **Extent of space** in direction i

$$x_i(t)^2 \equiv \left\langle \frac{1}{n} \text{Tr}(\bar{A}_i)(t)^2 \right\rangle ,$$

- In a thermal state there is spontaneous symmetry breaking: $SO(9) \rightarrow SO(6) \times SO(3)$: three dimensions of space become larger, the others are confined.
[J. Nishimura and G. Vernizzi, JHEP **0004**, 015 (2000);
]S.-W. Kim, J. Nishimura and A. Tsuchiya, Phys. Rev. Lett. **109**, 011601 (2012)]

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Emergent Metric from Matrix Theory

S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468

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Conclusions

- Eigenvalues of A_0 become **emergent time**, continuous in $N \rightarrow \infty$ limit.
- Work in the basis in which A_0 is diagonal: pick n (**comoving spatial coordinate**) and consider the block matrix $\tilde{A}_i(t)$.
- **Physical distance** between $n_i = 0$ and n_i (**emergent space**):

$$l_{phys,i}^2(n_i, t) \equiv \left\langle \text{Tr}(\tilde{A}_i(t))^2 \right\rangle ,$$

- $l_{phys,i}(n_i) \sim n_i$ (for $n_i < n_c$)
- **Emergent infinite and continuous space** in $N \rightarrow \infty$ limit.
- **Emergent metric** (S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468).

$$g_{ii}(n_i)^{1/2} = \frac{d}{dn_i} l_{phys,i}(n_i)$$

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No Flatness Problem in Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468

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Emergent metric:

$$g_{ii}(n_i)^{1/2} = \frac{d}{dn_i} l_{phys,i}(n_i)$$

Result:

$$g_{ii}(n_i, t) = \mathcal{A}(t) \delta_{ii} \quad i = 1, 2, 3$$

$SO(3)$ symmetry \rightarrow

$$g_{ij}(\mathbf{n}, t) = \mathcal{A}(t) \delta_{ij} \quad i = 1, 2, 3$$

\rightarrow spatially flat.

Note: Local Lorentz invariance emerges in $N \rightarrow \infty$ limit.

No Flatness Problem in Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468

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Late Time Dynamics

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Conclusions

$$\mathcal{A}(t) \sim t^{1/2}$$

Note: no sign of a cosmological constant.

Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

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Conclusions

- We **assume** that the spontaneous symmetry breaking $SO(9) \rightarrow SO(3) \times SO(6)$ observed in the IKKT model also holds in the BFSS model.
- Using the Gaussian approximation method we have shown the existence of a symmetry breaking phase transition in the IKKT model (S. Brahma, RB and S. Laliberte, arXiv:2209.01255).
- **Thermal correlation functions** in the three large spatial dimensions calculated in the high temperature state of the BFSS model (following the formalism developed in String Gas Cosmology).
- \rightarrow curvature fluctuations and gravitational waves.

Matrix Theory Cosmology

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Matrix Theory Cosmology: Thermal Fluctuations

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

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Conclusions

- Start with the **BFSS partition function**.
- Note: $\frac{1}{T}$ correction terms in the BFSS action are crucial!
- Calculate matter correlation functions in the emergent phase.
- For fixed k , convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$.
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations.

Note: the matter correlation functions are given by partial derivatives of the **finite temperature partition function** with respect to T (density fluctuations) or R (pressure perturbations).

Extracting the Metric Fluctuations

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Conclusions

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta) \left((1 + 2\phi) d\eta^2 - [(1 - 2\phi)\delta_{ij} + h_{ij}] dx^i dx^j \right).$$

Inserting into the perturbed Einstein equations yields

$$\langle |\phi(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0_0(k) \delta T^0_0(k) \rangle,$$

$$\langle |h(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i_j(k) \delta T^i_j(k) \rangle.$$

Note: We assume the validity of the semi-classical Einstein equations in the far IR.

Extracting the Metric Fluctuations

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Computation of Fluctuations I

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

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Conclusions

$$P(k) = k^3 (\delta\Phi(k))^2 = 16\pi^2 G^2 k^2 T^2 C_V(R)$$

$$C_V(R) = \frac{\partial}{\partial T} E(R)$$

$$E = -\frac{\partial}{\partial \beta} \ln Z(\beta)$$

Matrix Theory Cosmology: Results

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

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Conclusions

Thermal fluctuations in the emergent phase →

- **Scale-invariant spectrum of curvature fluctuations**
- **With a Poisson contribution for UV scales.**
- **Scale-invariant spectrum of gravitational waves.**

→ BFSS matrix model yields emergent infinite space, emergent infinite time, emergent spatially flat metric and an emergent early universe phase with thermal fluctuations leading to scale-invariant curvature fluctuations and gravitational waves.

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Open Problems

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Conclusions

- Include the effects of the fermionic sector.
- Understand **phase transition** to the expanding phase of Big Bang Cosmology.
- Understand the **emergence of GR** in the IR.
- Spectral indices?
- What about Dark Energy?
- Emergent low energy effective field theory for localized excitations.

Plan

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Conclusions

- 1 Trans-Planckian Censorship
- 2 Scenarios for a Successful Early Universe Cosmology
 - Criteria and Scenarios
 - String Gas Cosmology
- 3 Emergent Metric Space-Time from Matrix Theory
- 4 Matrix Theory Cosmology
- 5 Conclusions

Conclusions

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Conclusions

- Inflation is **not** the only scenario of early universe cosmology consistent with current data.
- In light of the TCC and other conceptual problems **effective field theory models of inflation are not viable.**
- In light of the TCC and other conceptual problems **Dark Energy** cannot be a cosmological constant.
- We need to go **beyond point particle EFT** in order to describe the very early universe.

Conclusions

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Conclusions

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Conclusions

- **BFSS matrix model** is a proposal for a non-perturbative definition of superstring theory. Consider a **high temperature state** of the BFSS model.
- → **emergent time, space and metric**. Emergent space is **spatially flat** and infinite.
- **Thermal fluctuations** of the BFSS model → **scale-invariant spectra of cosmological perturbations and gravitational waves**.
- **Horizon problem, flatness problem and formation of structure problem** of Standard Big Bang Cosmology resolved **without requiring inflation**.
- Transition from an emergent phase to the radiation phase of expansion. **No cosmological constant**.

Computation of Fluctuations II

N. Kawahara, J. Nishumura and S. Takeuchi, arXiv:0710.2188

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Conclusions

$$E^2 = N^2 \langle \mathcal{E} \rangle_{BFSS}, \quad \mathcal{E} = -\frac{3}{4N\beta} \int_0^\beta dt \text{Tr}([X_i, X_j]^2)$$

- Insert Matsubara expansion of the matrices: leading term in the BFSS action in the high T limit is the IKKT action.
- Express expectation values in terms of IKKT expectation values

To next to leading order in $1/T$:

$$E^2 = \frac{3}{4} N^2 \chi_2 T - \frac{3}{4} N^4 \alpha \chi_1 T^{-1/2}$$

$$\chi_1 = \langle R^2 \rangle_{BFSS} T^{-1/2}$$

Relationship between IKKT Model and Type IIB String Theory

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Conclusions

Consider action of the Type IIB string theory in Schild gauge

$$S_{\text{Schild}} = \int d^2\sigma \alpha \left[\sqrt{g} \left(\frac{1}{4} \{X^\mu, X^\nu\} - \frac{i}{2} \bar{\psi} \Gamma^\mu \{X^\mu, \psi\} \right) + \beta \sqrt{g} \right].$$

$$\text{Partition function : } Z = \int \mathcal{D}\sqrt{g} \mathcal{D}X \mathcal{D}\psi e^{-S_{\text{Schild}}}.$$

$$\text{Correspondence : } \{, \} \rightarrow -i[,]$$

$$\int d^2\sigma \sqrt{g} \rightarrow \text{Tr}$$

Obtain grand canonical partition function of IKKT model.

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Conclusions

Starting point: finite temperature partition function:

$$Z(\beta) = \int \mathcal{D}A \mathcal{D}X_i e^{-S(\beta)}$$

Internal energy

$$E = -\frac{d}{d\beta} \ln Z(\beta)$$

$$E = -\frac{3}{4} \lambda^{-1} \frac{N}{\beta} \int_0^\beta dt \text{Tr}[X_i, X_j]^2$$

Matsubara expansion:

$$X_i = \sum_n X_i^n e^{i(2\pi n / \beta)t}$$

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Conclusions

Matsubara expansion of the action:

$$S_{BFSS} = S_0 + S_{kin} + S_{int}$$

At high temperature: S_{kin} and S_{int} suppressed compared to S_0 .

To next to leading order:

$$E \simeq \lambda^{-1} \frac{3N^2}{4} \chi_2 T - \lambda^{-1} \frac{3N^2}{4} \mathcal{O}(1) \chi_2 \chi_1 T^{-1/2}$$

where $\chi_1 \simeq R^2 \lambda^{4/3} T^{-1/2}$.

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Conclusions

- Derivative w.r.t. $T \rightarrow$ density fluctuations: both terms contribute.
- Derivative w.r.t. $R \rightarrow$ pressure fluctuations: only second term contributes.

Power spectrum $P(k)$ of density fluctuations: ($k = R^{-1}$)

- First term dominates in the UV: Poisson spectrum.
- Second term dominated in the IR: Scale-invariant spectrum.

$$P(k) = 16\pi^2 G^2 \lambda^{4/3} N^2 \mathcal{O}(1) \sim (l_s m_{pl})^{-4}$$

using the scaling $G^2 N^2 \lambda^{4/3} \sim (l_s m_{pl})^{-4}$.

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