Hybrid cosmological attractors and PBH

Andrei Linde

Plan:

1) Hybrid inflation and PBH: Garcia-Bellido, A.L. and Wands 1996

2) Cosmological attractors: Kallosh, A.L., Roest 2014

3) Hybrid cosmological attractors: A.L. and Kallosh 2022

4) Hybrid attractors and PBH: Braglia, A.L., Kallosh and Finelli 2022

Density Perturbations and Black Hole Formation in Hybrid Inflation

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(May 15, 1996)

We investigate the recently proposed hybrid inflation models with two stages of inflation. We show that quantum fluctuations at the time corresponding to the phase transition between the two inflationary stages can trigger the formation of a large number of inflating topological defects. In order to study density perturbations in these models we develop a new method to calculate density perturbations in a system of two scalar fields. We show that density perturbations in hybrid inflation models of the new type can be very large on the scale corresponding to the phase transition. The resulting density inhomogeneities lead to a copious production of black holes. This could be an argument against hybrid inflation models with two stages of inflation. However, we find a class of models where this problem can be easily avoided. The number of black holes produced in these models can be made extremely small, but in general it could be sufficiently large to have important cosmological and astrophysical implications. In particular, for certain values of parameters these black holes may constitute the dark matter in the universe. It is also possible to have hybrid models with two stages of inflation where the black hole production is not suppressed, but where the typical masses of the black holes are very small. Such models lead to a completely different thermal history of the universe, where post-inflationary reheating occurs via black hole evaporation.

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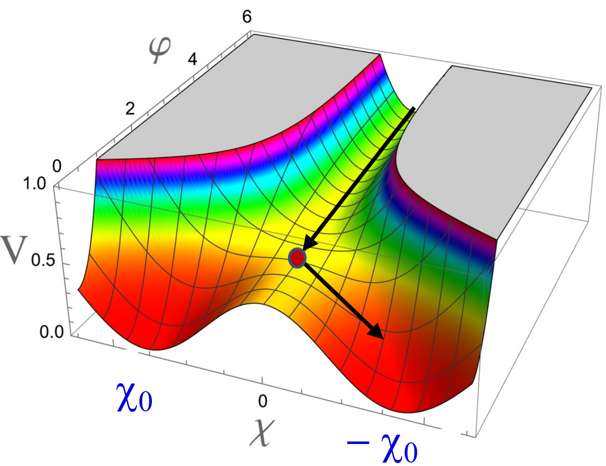
Hybrid inflation and PBH

Hybrid Inflation

$$V(\phi,\chi) = M^2 \frac{(\chi^2-\chi_0^2)^2}{4\chi_0^2} + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\chi^2$$



For $\chi = 0$, it is just the uplifted quadratic potential

For $\chi_0 << 1$ rapid waterfall at $\phi > \phi_c = M/g$

For $\chi_0 > 1$ the waterfall becomes inflationary

Consider first the potential at $\phi = 0$: $V(\chi) = M^2 \frac{(\chi^2-\chi_0^2)^2}{4\chi_0^2}$

If $\chi_0 > 1$, this is an inflationary Higgs-type potential, with eternal inflation at the top (see next page). Eternal inflation implies the amplitude of adiabatic perturbations O(1), resulting in PBH formation.

In fact, the condition $\chi_0 > 1$ is too strong, because large perturbations are generated already on the way towards $\phi = 0$.

Garcia-Bellido, A.L, and Wands 1996

$$V(\chi) = \frac{M^2}{4\chi_0^2}(\chi^2 - \chi_0^2)^2$$

$$V(0) = \frac{M^2}{4}\chi_0^2, \qquad V_\chi(0) = 0, \qquad V_{\chi,\chi}(0) = -M^2$$

$$\frac{|V_{\chi,\chi}(\phi)|}{V(\phi)}_{\phi<\phi_c} < \frac{|V_{\chi,\chi}(0)|}{V(0)} = \frac{4}{\chi_0^2}$$

Thus, we have (eternal) inflation at the top of the Higgs potential if $\chi_0 \gtrsim 2$

$$A_s = \frac{V^3}{12\pi^2 V_\chi^2}$$

For $\chi_0 \gtrsim 2$ hybrid inflation is eternal, and the amplitude of the fluctuations produced at the ridge of the potential with $V_\chi = 0$ is extremely large.

By considering $\chi_0 < 1$ one can make the amplitude of perturbations smaller than O(1), but still sufficiently large for PBH production. (Note that $V_{\chi,\chi}(\phi) <$ $V_{\chi,\chi}(0)$ for $0 < \phi < \phi_c$)

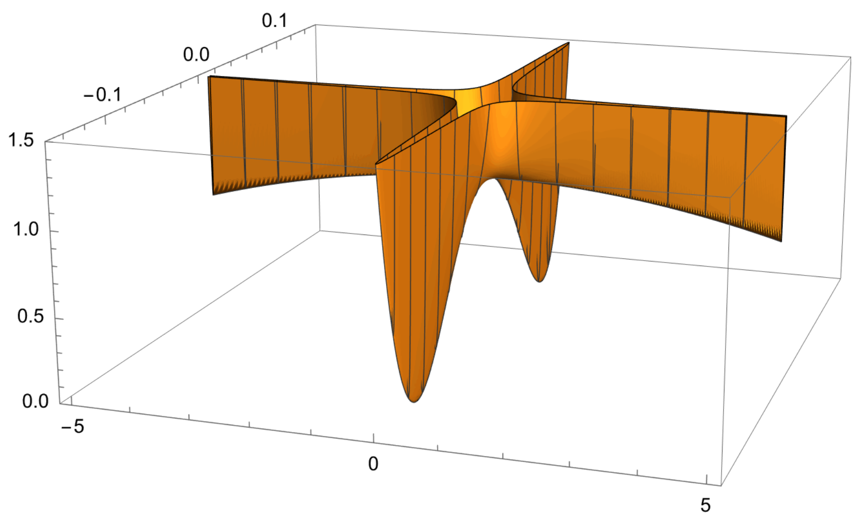
In hybrid inflation we deal with a two-field evolution, so the full theory of what is going on is more complicated, but the simple argument given above gives a qualitatively correct result.

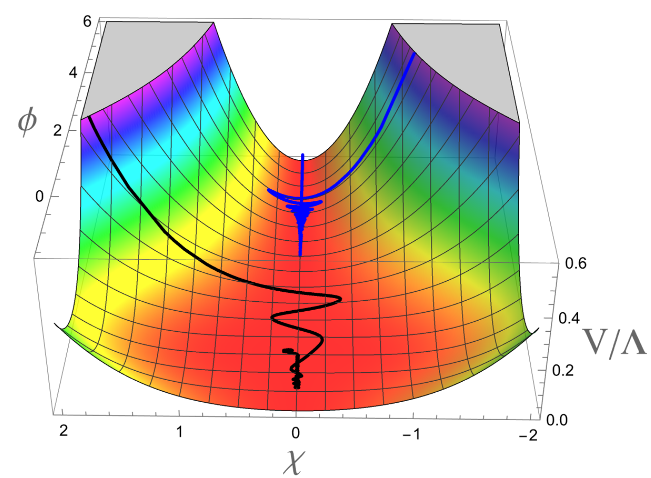
However, in the simplest hybrid inflation scenario with a quadratic inflaton potential one has $n_s > 1$, which contradicts WMAP and Planck results.

To get $n_s < 1$, Clesse and Bellido in 2015 proposed to change the inflaton potential, making it tachyonic.

$$V(\phi,\psi) = \Lambda\left[\left(1-\frac{\psi^2}{M^2}\right)^2 + \frac{2\phi^2\psi^2}{M^2\phi_c^2} + \underline{\frac{(\phi-\phi_c)}{\mu_1} - \frac{(\phi-\phi_c)^2}{\mu_2^2}}\right]$$

Unfortunately, extreme fine-tuning of initial conditions is required to have inflation and avoid the tachyonic instability in this scenario.



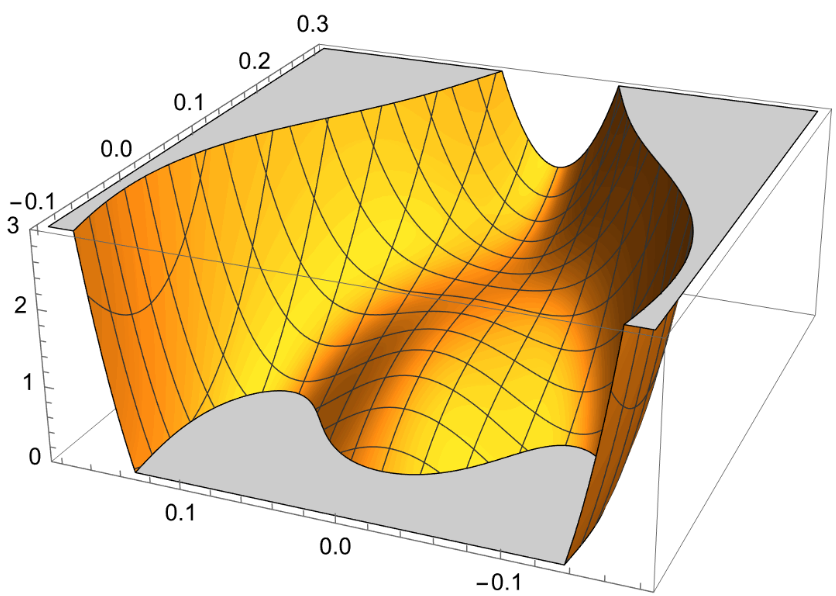


One may try to add higher order terms to the Clesse-Bellido potential

Yuichiro Tada and Masaki Yamada 2304.01249

$$V(\phi,\psi) = \Lambda\left[\left(1-\frac{\psi^2}{M^2}\right)^2 + \frac{2\phi^2\psi^2}{M^2\phi_c^2} + \underline{\frac{(\phi-\phi_c)}{\mu_1} - \frac{(\phi-\phi_c)^2}{\mu_2^2} + \frac{(\phi-\phi_c)^3}{\mu_3^3}}\right]$$

However, the resulting potential with the parameters used in 2304.01249 does not have a minimum at V = 0, so it requires additional modifications



All of these models are symmetric with respect to the change of sign of the Higgs field ψ. This results in production of superheavy topological defects, which requires additional modifications of these models.

To resolve all of these problems, we turned out to the α-attractor generalizations of the hybrid inflation scenario.

Braglia, A.L., Kallosh and Finelli 2022

T-models of α-attractors

Kallosh, AL, Roest 2014

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{(\partial_\mu \phi)^2}{2\left(1-\frac{\phi^2}{6\alpha}\right)^2} - V(\phi)$$

In canonical variables

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{(\partial_\mu \varphi)^2}{2} - V\left(\sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)$$

Asymptotically at large values of the inflaton

$$V(\varphi) = V_0 - 2\sqrt{6\alpha}\,V_0'\ e^{-\sqrt{\frac{2}{3\alpha}}\varphi}$$

Here $V_0' = \partial_\phi V|_{\phi=\sqrt{6\alpha}}$ This factor can be absorbed in the redefinition (shift) of the field. **Therefore, at small α, values of n_s and r depend only on V_0 and α**, not on the shape of V(ϕ).

$$n_s = 1 - \frac{2}{N_e}\ , \qquad r = \frac{12\alpha}{N_e^2}$$

E-models of α-attractors

Kallosh, AL, Roest 2014

Start with the model

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{3\alpha}{4}\frac{(\partial\rho)^2}{\rho^2} - V(\rho)$$

Switch to canonical variables

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{1}{2}(\partial\varphi)^2 - V(e^{-\sqrt{\frac{2}{3\alpha}}\varphi}).$$

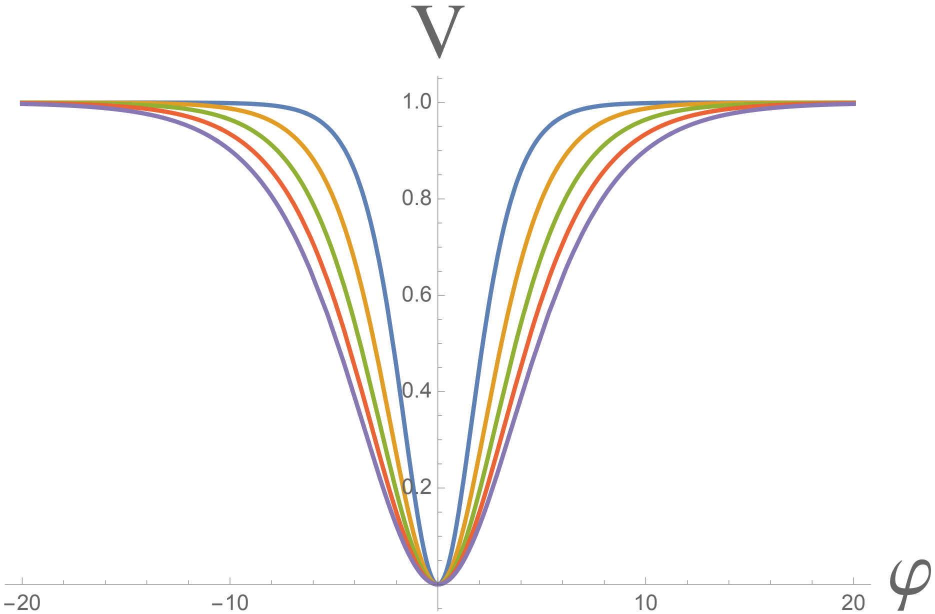
In particular, for $V(\rho) = V_0(1-\rho)^2$ the potential becomes

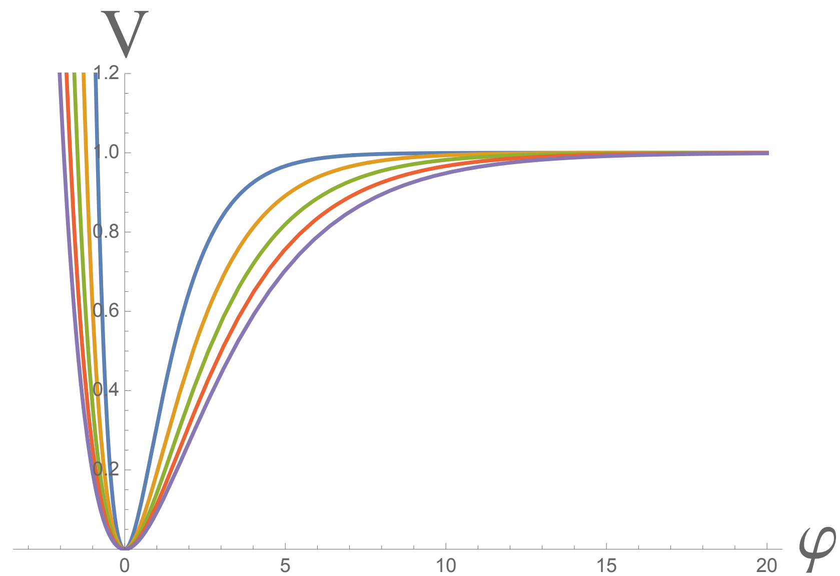
$$V = V_0\Big(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\Big)^2$$

This model (E-model) coincides with the Starobinsky model for α =1.
Just as T-models, these models predict

$$n_s = 1 - \frac{2}{N_e}\,, \qquad r = \frac{12\alpha}{N_e^2}$$

Plateau potentials of α-attractors





$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \;\; - \alpha\mu^2\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)^2$$

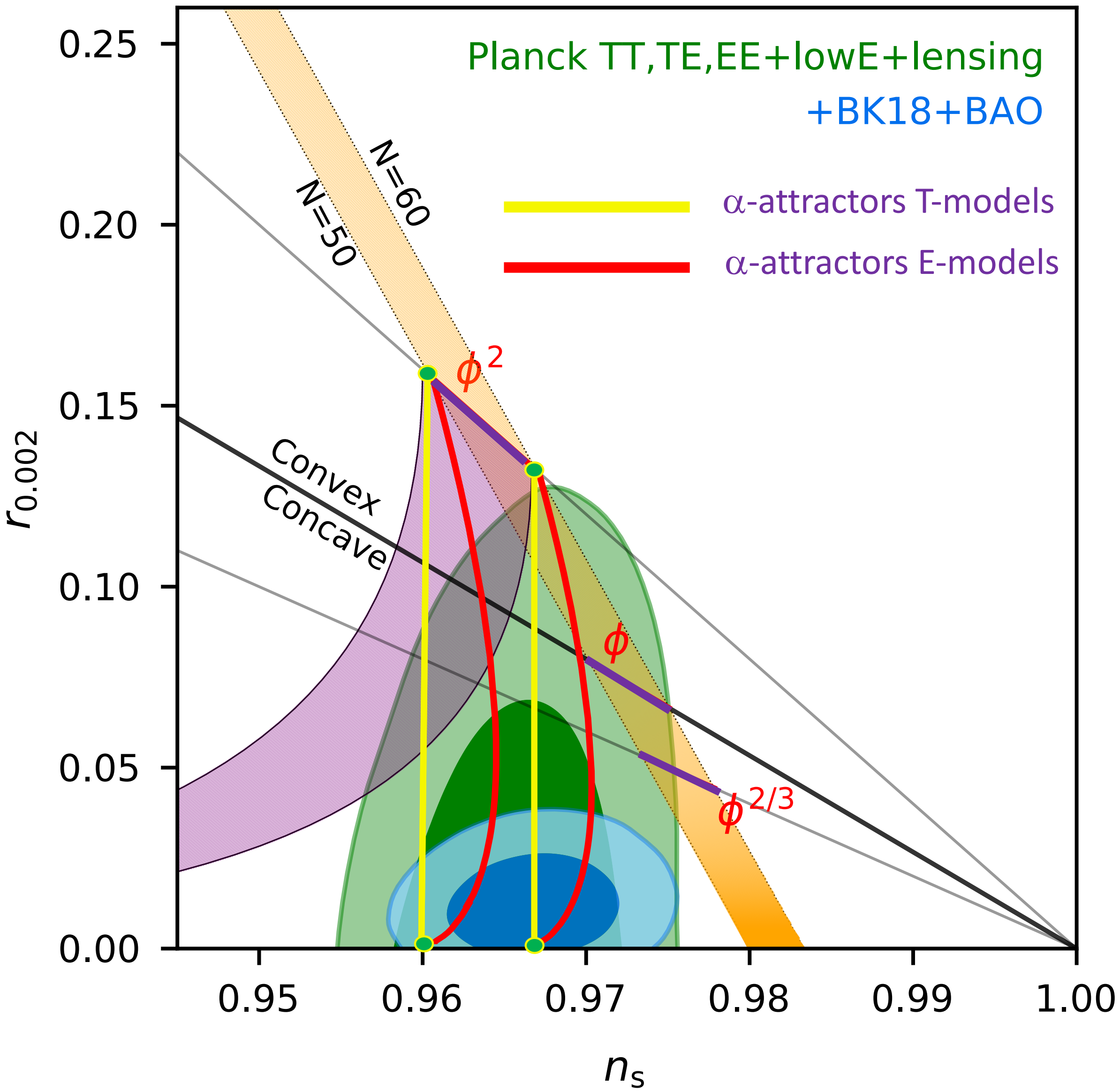
Simplest T-model

$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \;\; - \alpha\mu^2\left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\right)^2$$

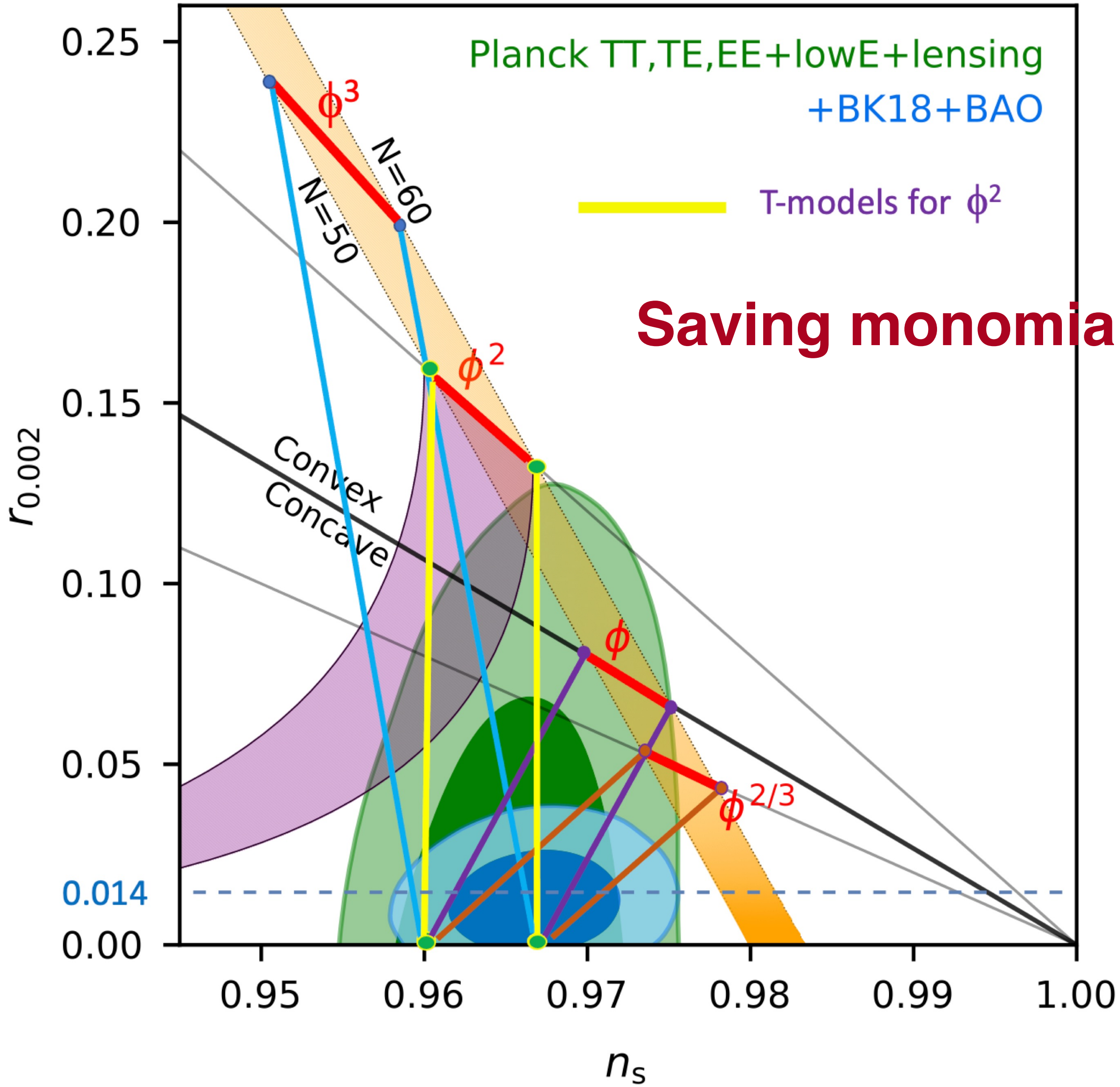
Simplest E-model

Planck2018 – BICEP/Keck2021 constraints

Kallosh, AL [2110.10902]



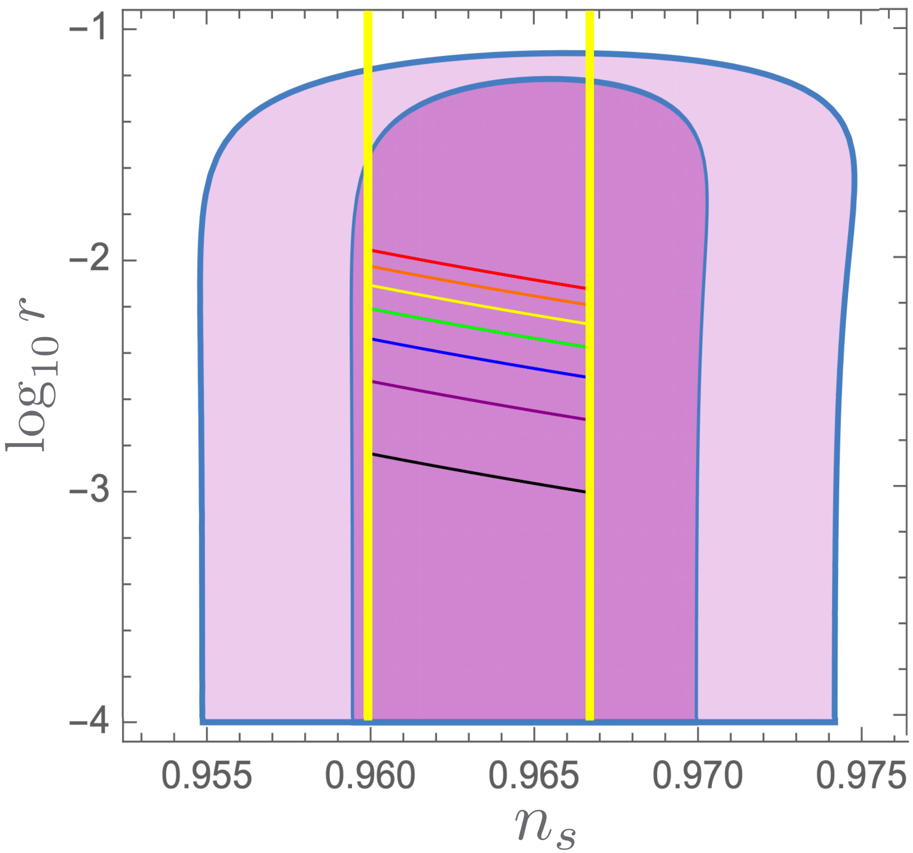
Planck2018 – BICEP/Keck2021 constraints



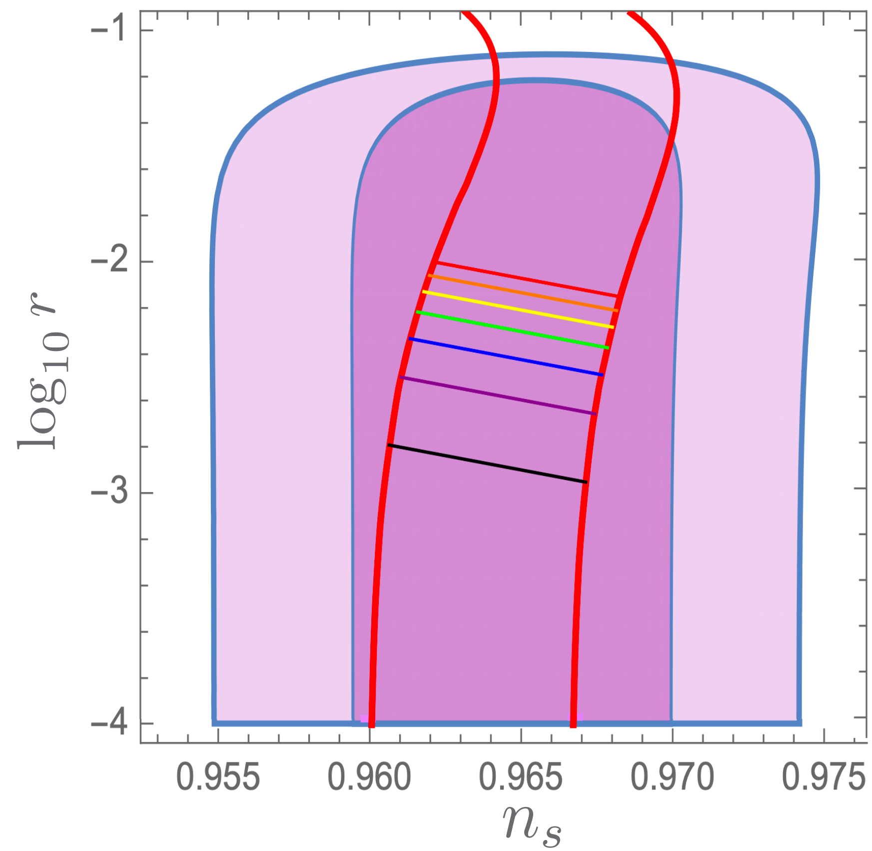
Saving monomial potentials

Benchmarks for T-models and E-models

T-models $V_T = V_0 \tanh^2 \frac{\varphi}{\sqrt{6\alpha}}$



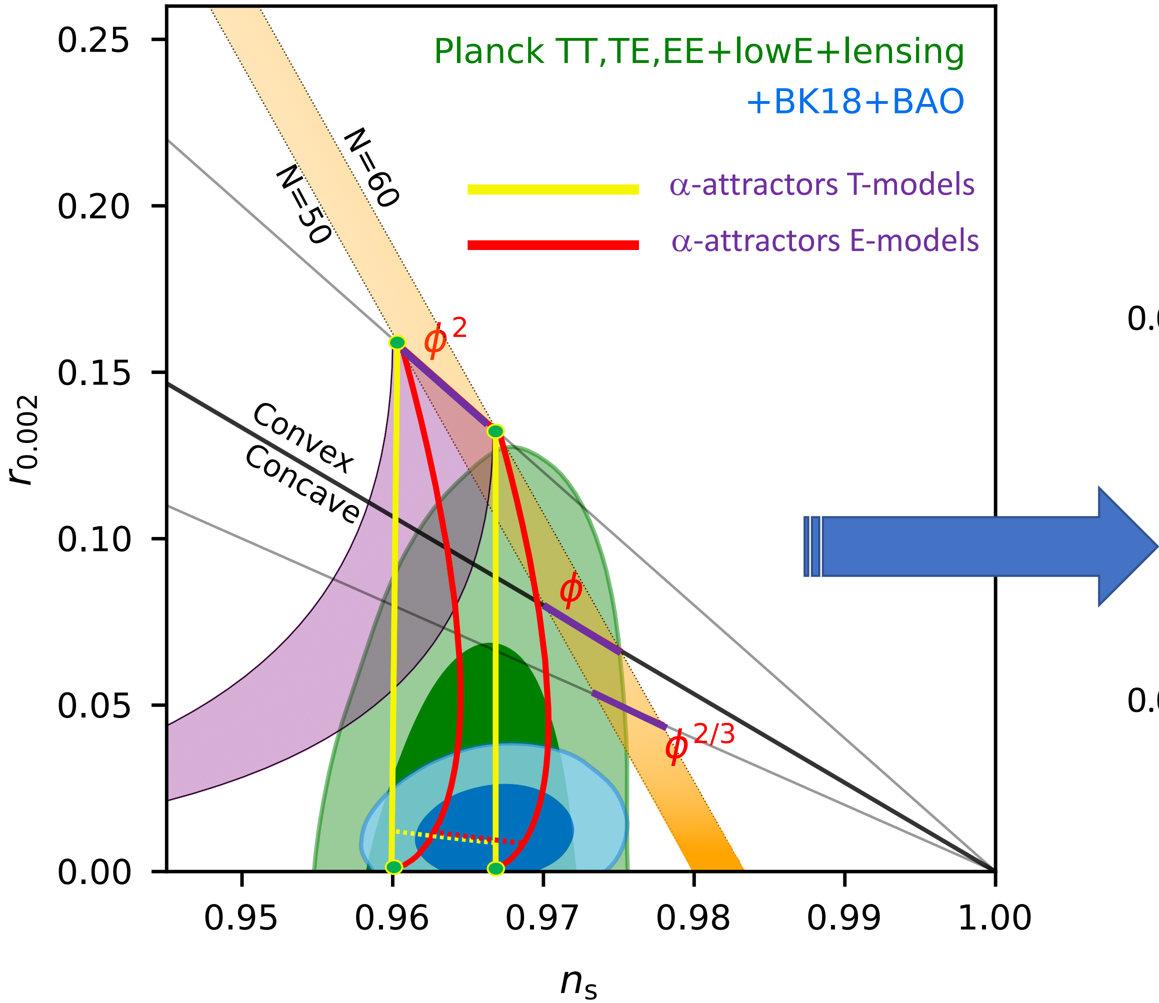
E-models $V_E = V_0 \left(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\right)^2$



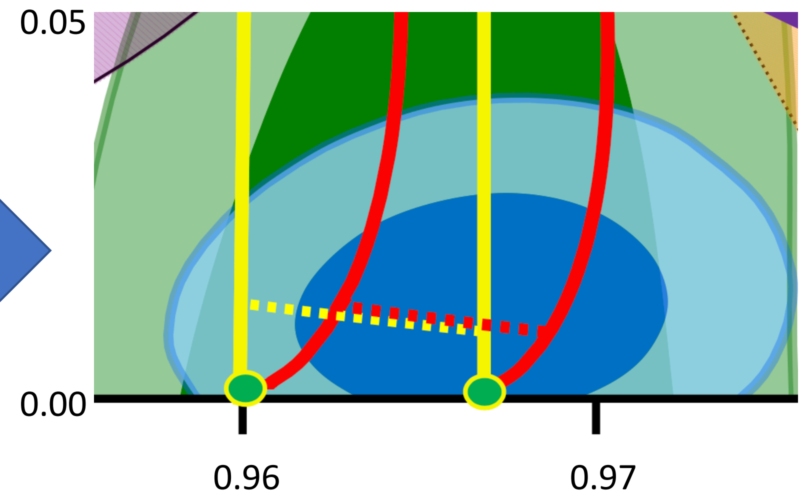
String theory interpretation of 7 discrete targets for α-attractors

Ferrara, Kallosh 1610.04163. Kallosh, A.L., Wrase, Yamada 1704.04829

BICEP/Keck2021 do not claim a discovery of the gravitational waves. The error bars of their result $r_{0.05} = 0.014^{+0.010}_{-0.011}$ are too large, $\sigma(r) = 0.009$. However, it is quite intriguing that the yellow and red dashed lines, which show the predictions of the largest option α = 7/3, go straight through the center of the dark blue ellipse favored by Planck/BICEP/Keck data.



BICEP/Keck hope to reach σ(r) = 0.003 within 5 years.



LiteBIRD

[Probing Cosmic Inflation with the LiteBIRD Cosmic Microwave Background Polarization Survey](2202.02773)

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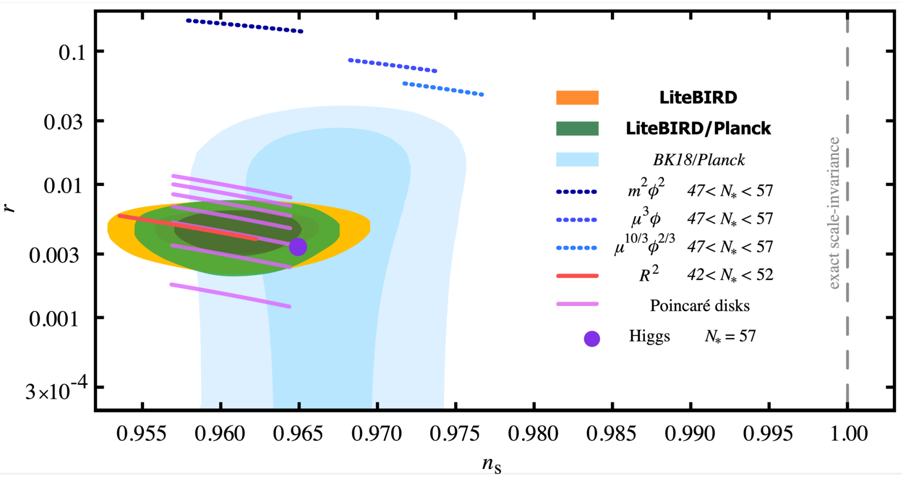
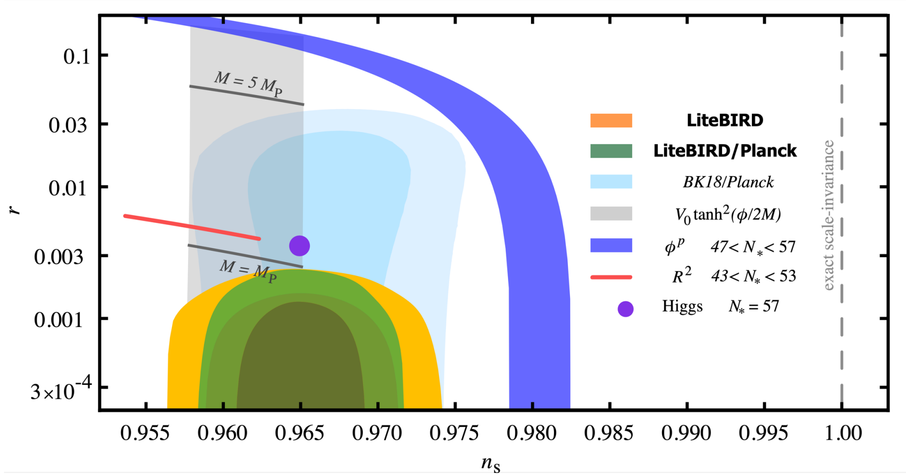




Are there any good models for the right side of the blue area favored by BICEP/Keck ??

General pole inflation

Galante, Kallosh, AL, Roest 2014

Start with the model

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{a_q}{2}\frac{(\partial\rho)^2}{\rho^q} - V(\rho)$$

For q = 2 we have α-attractors

For q > 2 and small r one has an attractor regime with

$$n_s = 1 - \frac{\beta}{N_e}\,, \qquad \beta = \frac{q}{q-1}$$

Some of these models have interpretation in terms of Dp brane inflation (KKLTI models) Dvali, Tye, 1998, Kachru, Kallosh, A.L., Maldacena, McAllister, Trivedi 2003, Kallosh, A.L. Yamada 1811.01023

$$V_{Dp-\bar{D}p} \sim \frac{\varphi^k}{m^k + \varphi^k} = \Big(1 + \frac{m^k}{\varphi^k}\Big)^{-1}, \qquad k = 7 - p = \frac{2}{2-q}$$

General pole inflation

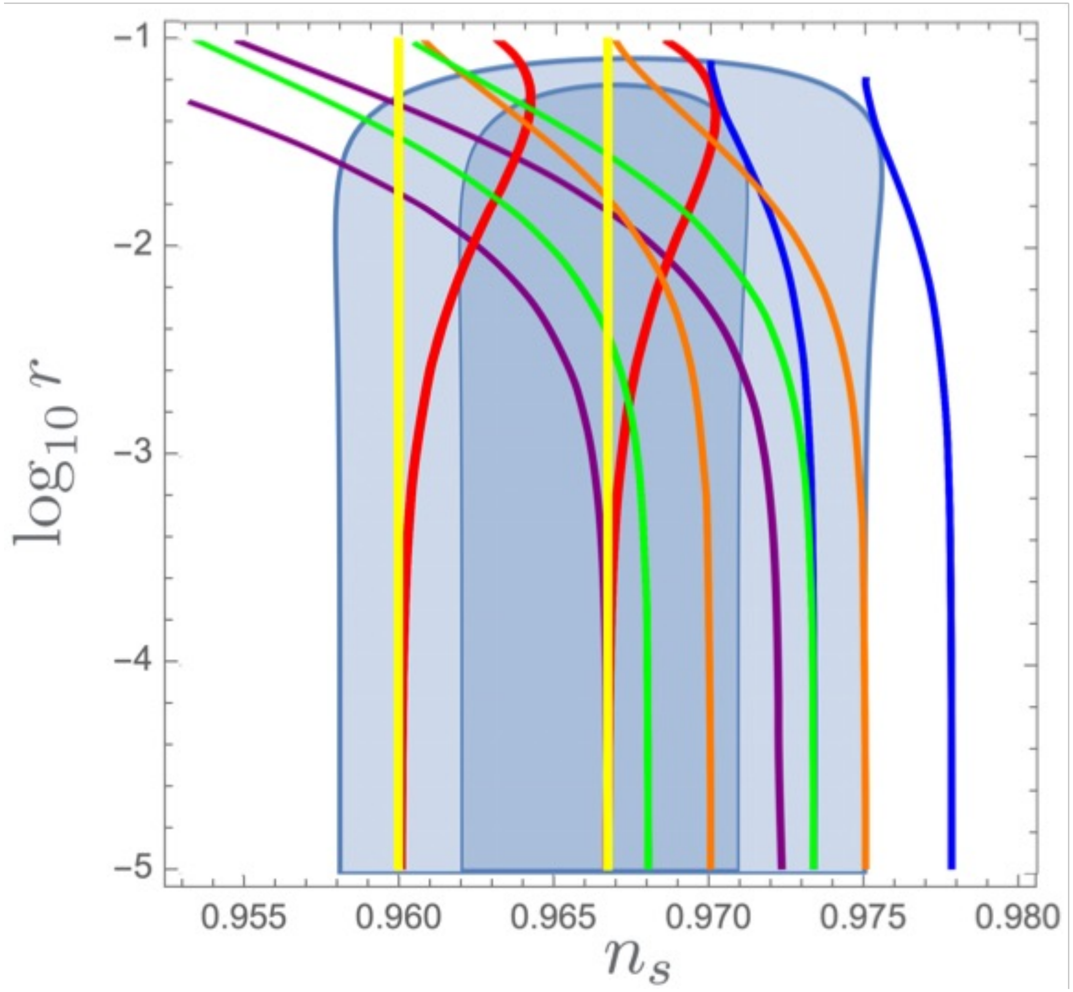
In particular, for D3 branes and small m one has KKLTI potential

$$V = V_0 \, \frac{\varphi^4}{m^4 + \varphi^4} \,, \qquad n_s = 1 - \frac{5}{3N} \,, \qquad r = \frac{4m^{\frac{4}{3}}}{(3N)^{\frac{5}{3}}}$$

For D5 branes and small m one has

$$V = V_0 \, \frac{\varphi^2}{m^2 + \varphi^2} \,, \qquad n_s = 1 - \frac{3}{2N_e} \,, \qquad r = \frac{\sqrt{2}\, m}{N_e^{\frac{3}{2}}}$$

The last potential emerges also in the model of two interacting fields with the flattening mechanism introduced by Stewart in 1995, and by Dong, Horn, Silverstein and Westphal in 2011



From left to right, we show predictions of T-models and E-models (yellow and red lines for N_e = 50, 60) and of Dp brane inflation with p = 3, 4, 5, 6 (purple, green, orange and blue lines). These models, belonging to the general class of pole inflation, can describe gravitational waves all the way down to r =0.

Polynomial α-attractors

Kallosh, AL, 2202.06492

Start with the model

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{3\alpha}{4}\frac{(\partial\rho)^2}{\rho^2} - V(\rho)$$

The usual assumption was that V(ρ) and its derivatives is not singular at ρ = 0. Now we will relax this requirement. *We will still require V(ρ) to be non-singular, but we will allow its derivatives to be singular*.

Example: For $V(\rho) = V_0 \frac{\ln^2 \rho}{\ln^2 \rho + c^2}$ the inflaton potential in canonical variables is

$$V = V_0 \frac{\varphi^2}{\varphi^2 + \mu^2} \quad \text{where} \quad \mu^2 = \frac{3\alpha}{2} c^2$$

For $V = V_0 \frac{\ln^4 \rho}{\ln^4 \rho + c^4}$ the potential becomes the KKLTI potential

$$V = V_0 \frac{\varphi^4}{\varphi^4 + \mu^4}$$

Polynomial α-attractors

Kallosh, AL, 2202.06492

Using similar but different functions V(ρ) one can obtain a broad class of potentials approaching a plateau at $\varphi \to \infty$ as inverse powers of the field.

$$V = V_0(1 - \frac{\mu^k}{\varphi^k} + ...)$$

Here k is an arbitrary positive number (not necessarily integer). Independently of the detailed structure of the potential, one finds

$$n_s = 1 - \frac{2}{N_e}\frac{k+1}{k+2}$$

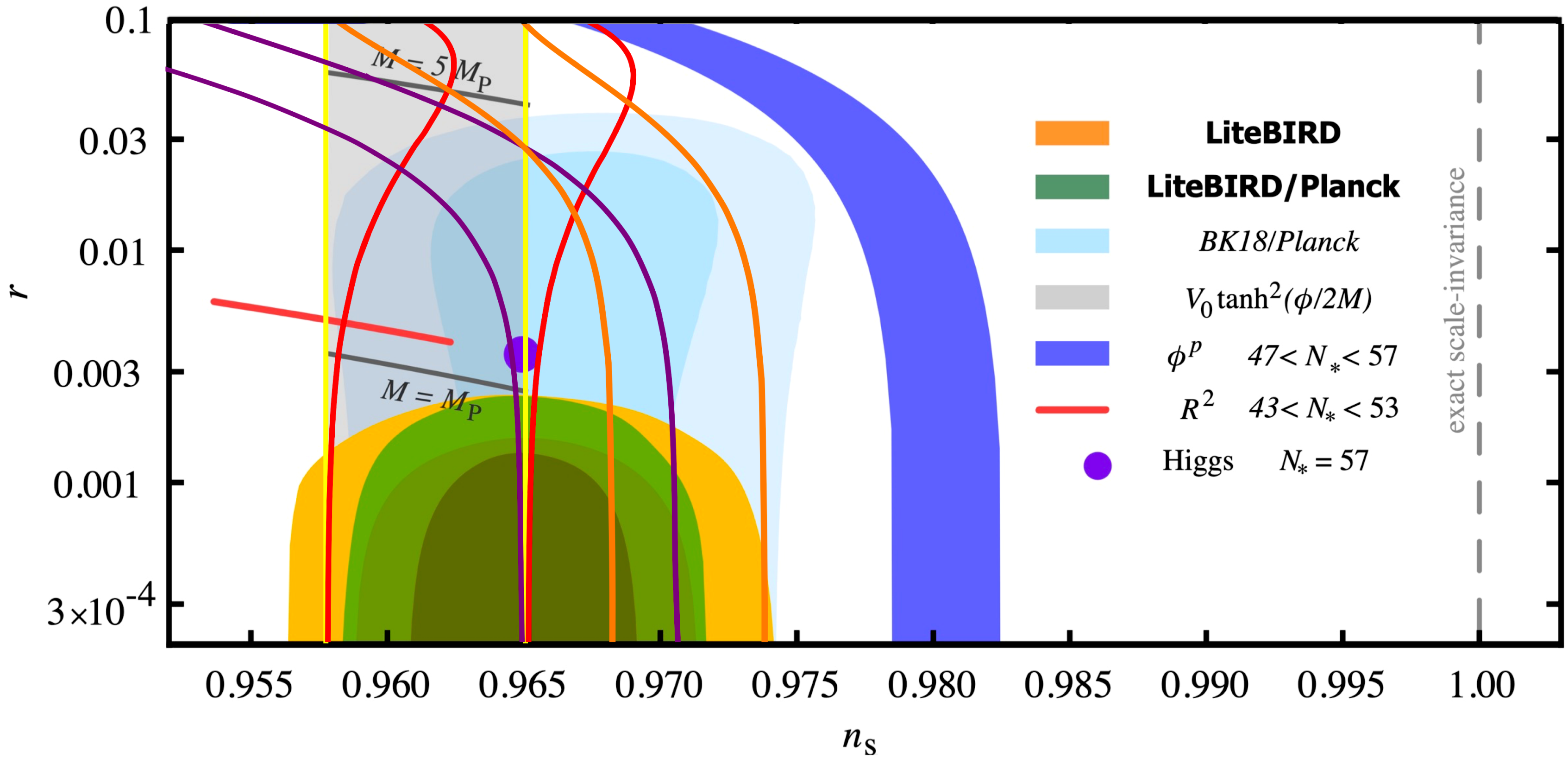
For any positive k, one has

$$n_s > 1 - \frac{2}{N_e}$$

In the limit of small k, one has

$$n_s \to 1 - \frac{1}{N_e}$$

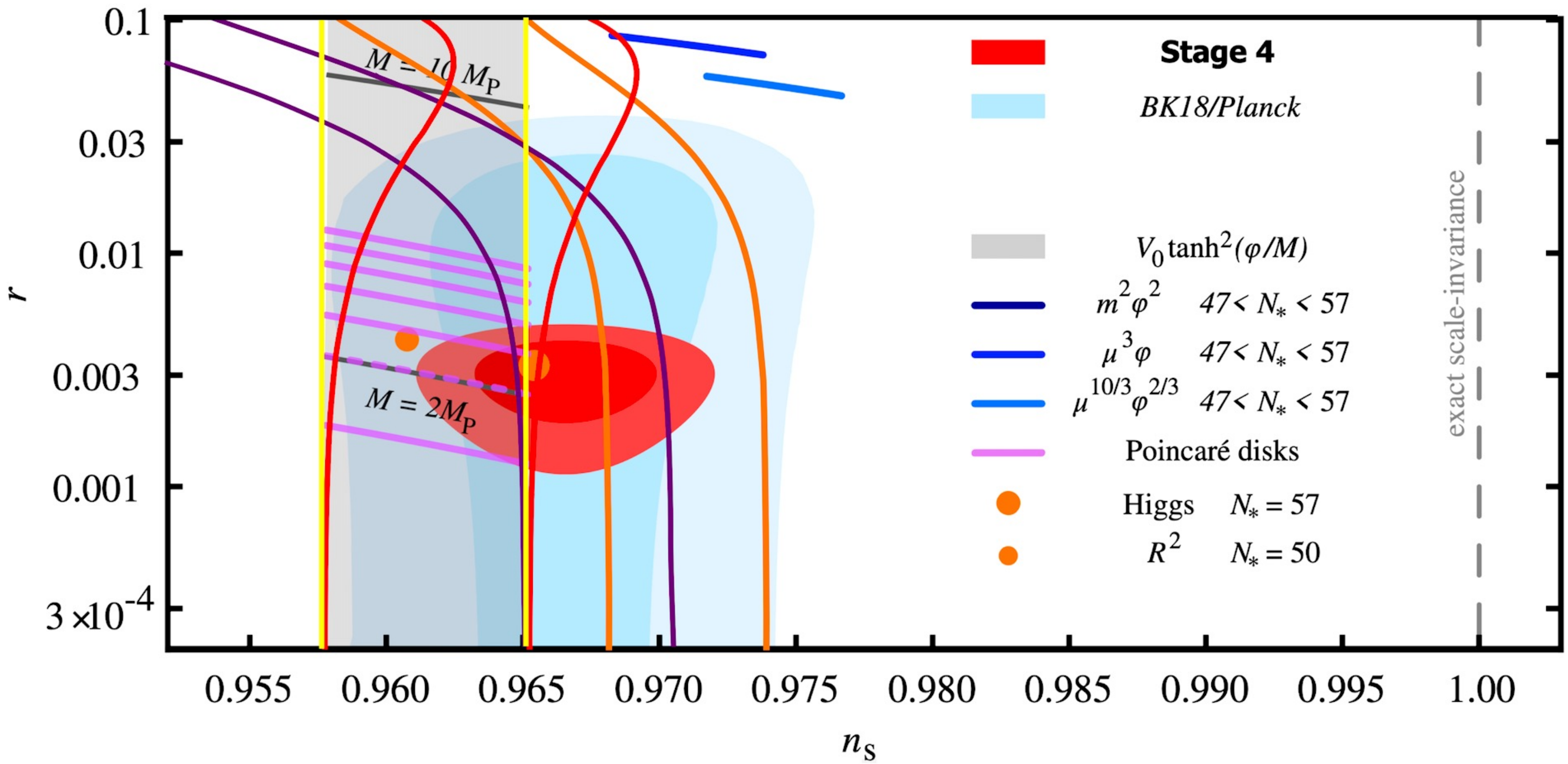
LiteBIRD



The gray area shows predictions of T-models. The two red lines show predictions of E-models. The purple and orange lines correspond to the polynomial α-attractors $\frac{\varphi^4}{\varphi^4+\mu^4}$ and $\frac{\varphi^2}{\varphi^2+\mu^2}$. These models completely cover the dark blue area favored by Planck/BICEP/Keck

CMB-S4

Snowmass2021 Cosmic Frontier: CMB Measurements White Paper



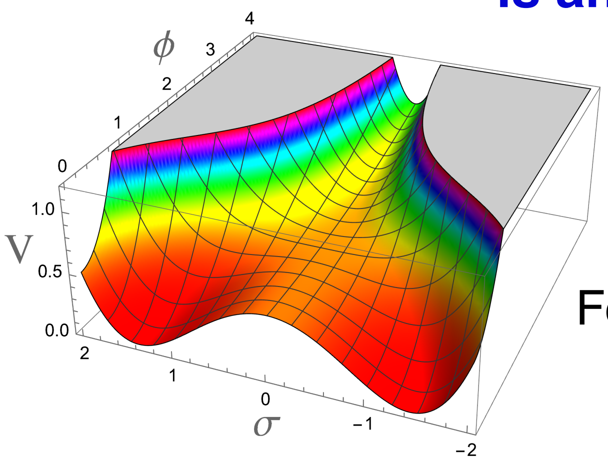
The gray area shows predictions of T-models. The two red lines show predictions of E-models. The purple and orange lines correspond to the polynomial α-attractors $\frac{\varphi^4}{\varphi^4+\mu^4}$ and $\frac{\varphi^2}{\varphi^2+\mu^2}$. These models completely cover the dark blue area favored by Planck/BICEP/Keck

But what if H_0 problem changes everything?

There are many attempts to resolve the disagreement between Planck results and supernova data by considering early dark energy, new early dark energy, etc. Some of these attempts favor large values of n_s, all the way to $n_s = 1$. While this is an exotic possibility, it is better to be prepared and consider maximally flexible inflationary models. Can we get $n_s = 1$ from cosmological attractors?

There is a special class of inflationary models where $n_s = 1$ is an attractor point: Hybrid Inflation.

AL 1991, 1994



$$V(\sigma,\phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2$$

For σ = o, it is just the quadratic potential uplifted by $M^4/4\lambda$

$$n_s = 1 - 3\left(\frac{V'}{V}\right)^2 + 2\frac{V''}{V}$$

By increasing the uplifting term $V_{\text{uplift}} = M^4/4\lambda$ one can increase V without changing any derivatives of V. In the large uplift limit, we have a universal attractor prediction $n_s = 1$ for any V growing at large ϕ

Hybrid α attractors: Two attractor regimes

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{(\partial_\mu \phi)^2}{2\left(1-\frac{\phi^2}{6\alpha}\right)^2} - \frac{(\partial_\mu \sigma)^2}{2\left(1-\frac{\sigma^2}{6\beta}\right)^2} - V(\sigma,\phi)$$

$$V(\sigma,\phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2$$

Just as in all α-attractors, we have a universal large N attractor prediction

$$n_s = 1 - \frac{2}{N_e}\ , \qquad r = \frac{12\alpha}{N_e^2}$$

However, if uplift is very large, the large N limit is reached only for N >> 60. Then for N ~ 60 one has a hybrid attractor large uplift prediction $n_s = 1$.

The value of N where the large N limit is reached depends on the relation between V_{uplift} and m^2. As a result, by changing V_{uplift} one can obtain any value of n_s in the range between the two attractor predictions

$$1 - \frac{2}{N_e} < n_s < 1$$

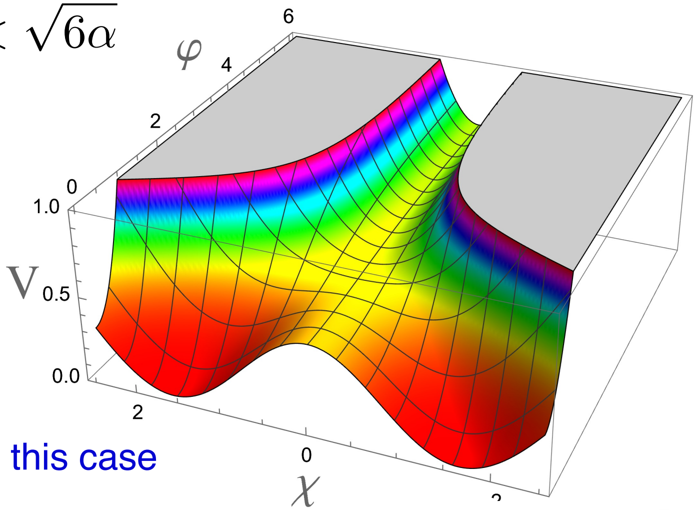
Kallosh, AL 2204.02425

Hybrid attractors are more complicated than the simplest α-attractors discussed in this talk. However, there are some situations where flexibility of theoretical models may be desirable.

Hybrid α attractors: New possibilities

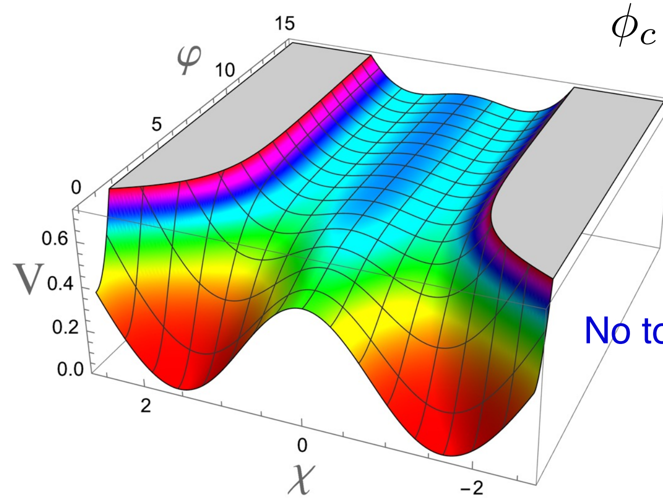
$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{(\partial_\mu \phi)^2}{2\left(1-\frac{\phi^2}{6\alpha}\right)^2} - \frac{(\partial_\mu \sigma)^2}{2\left(1-\frac{\sigma^2}{6\beta}\right)^2} - V(\sigma,\phi)$$

$$\phi_c = \frac{M}{g} < \sqrt{6\alpha}$$

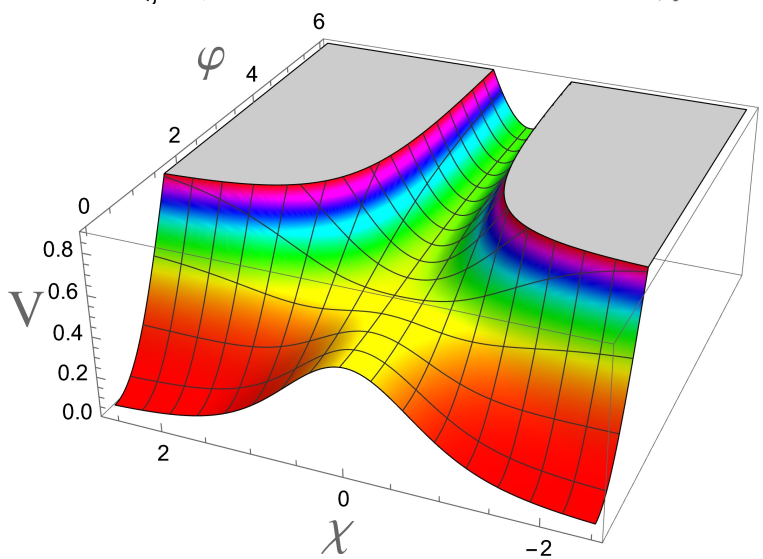


We will study this case

$$\phi_c = \frac{M}{g} > \sqrt{6\alpha}$$



No topological defects



$$\sigma_0 = \frac{M}{\sqrt{\lambda}} > \sqrt{6\beta}$$

Quintessential inflation,
or dark energy

Hybrid α attractors and PBH

Braglia, A.L., Kallosh and Finelli 2022

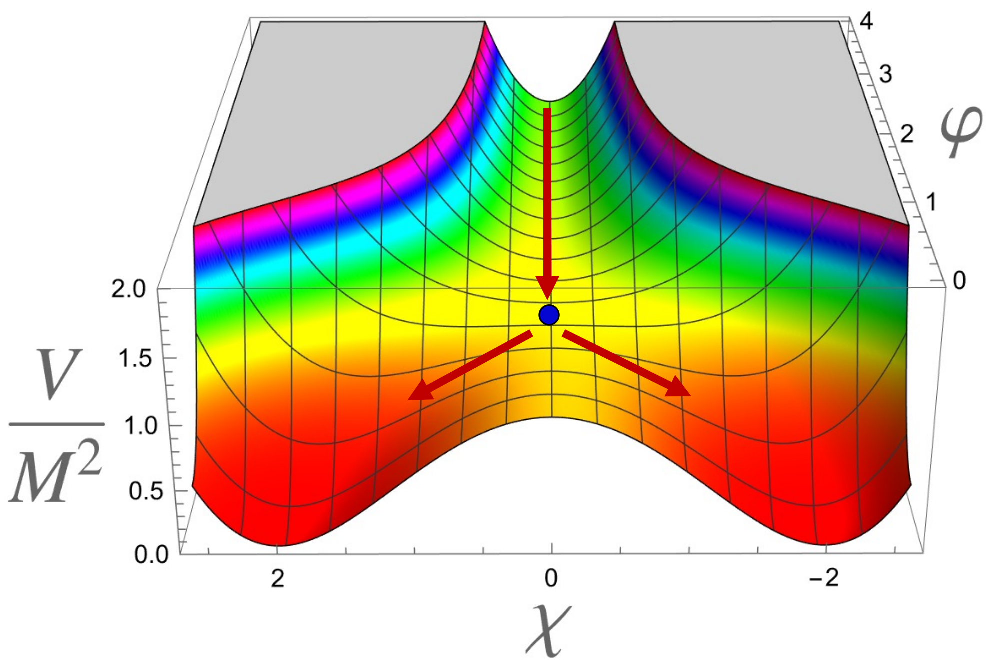
$$V(\chi,\phi) = M^2\left[\frac{(\chi^2-\chi_0^2)^2}{4\chi_0^2} + \frac{\tilde{m}^2}{2}\phi^2 + \frac{\tilde{g}^2}{2}\phi^2\chi^2 + d\,\chi\right]$$

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{(\partial_\mu\phi)^2}{2\left(1-\frac{\phi^2}{6\alpha}\right)^2} - \frac{(\partial_\mu\chi)^2}{2} - V(\chi,\phi)$$

Keep χ canonical for simplicity

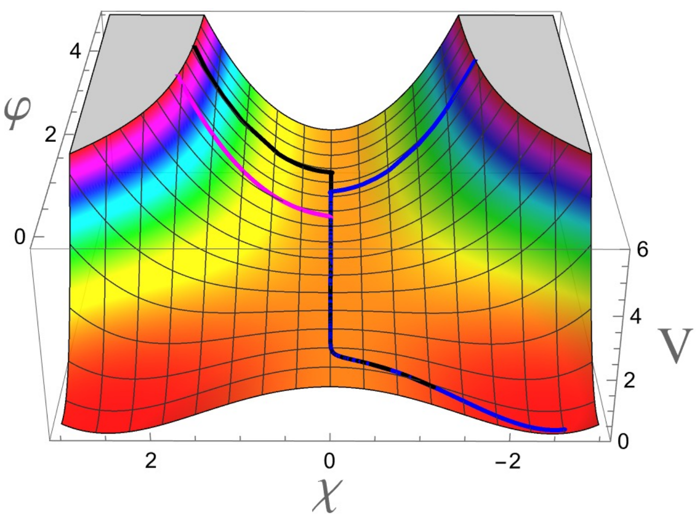
In canonical variables

$$V(\chi,\varphi) = M^2\left[\frac{(\chi^2-\chi_0^2)^2}{4\chi_0^2} + 3\alpha(\tilde{m}^2+\tilde{g}^2\chi^2)\tanh^2\frac{\varphi}{\sqrt{6\alpha}} + d\,\chi\right]$$

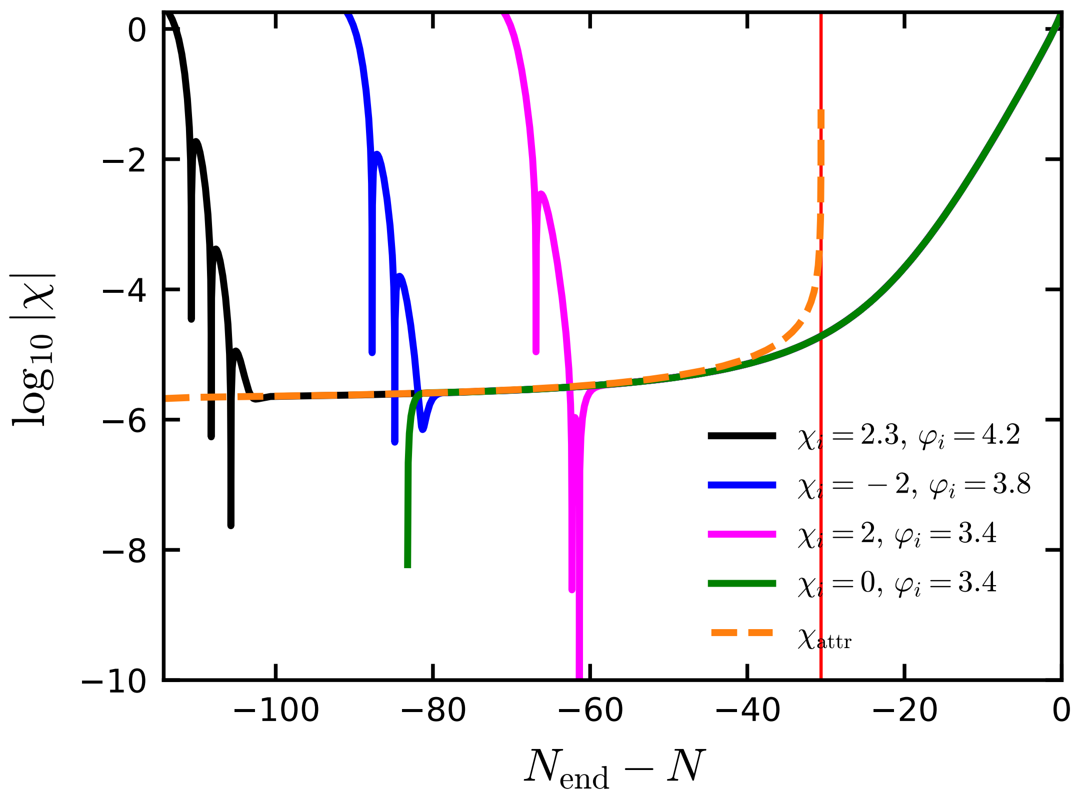


Initial conditions and inflation

Classical evolution with different initial conditions



Orange line shows evolution with an account taken of quantum fluctuations



The results are very stable with respect to the choice of initial conditions

As before, consider first the case $\phi = 0$

$$V(\chi) = M^2 \left[\frac{(\chi^2 - \chi_0^2)^2}{4\chi_0^2} + d\,\chi \right]$$

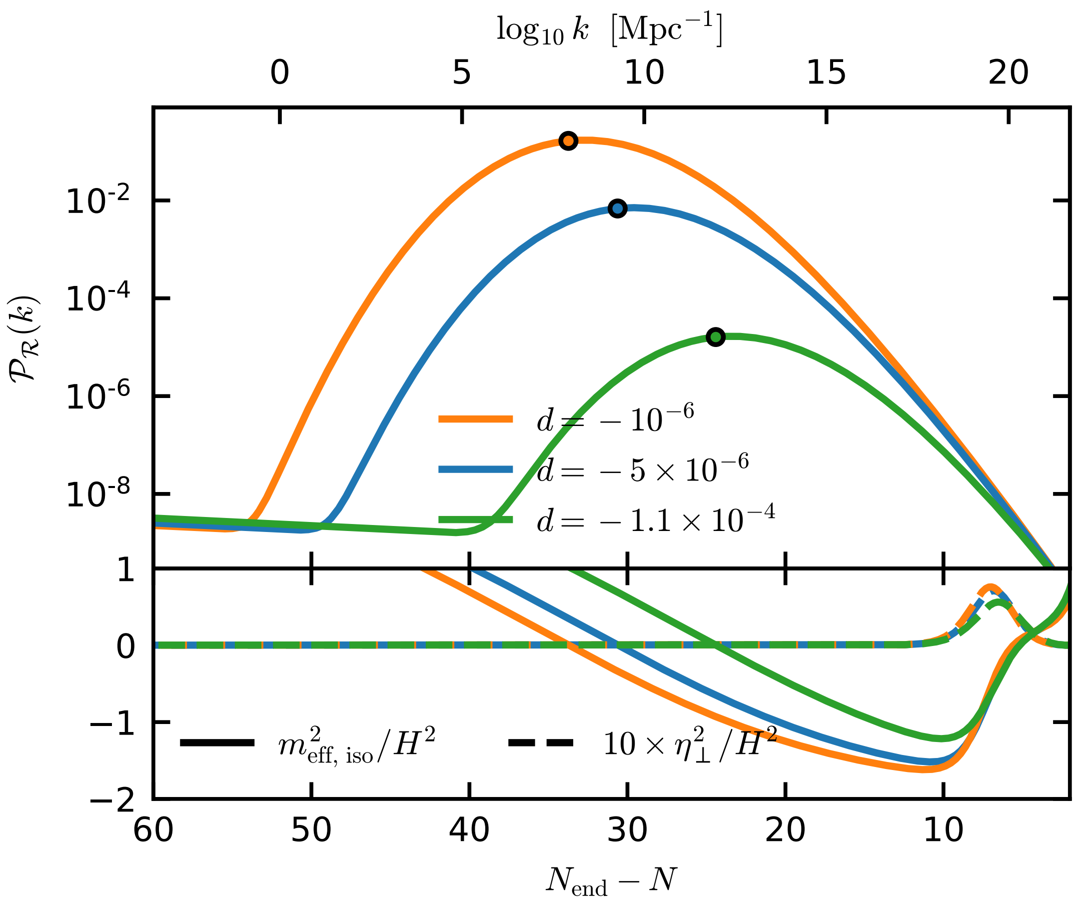
$$V(0) = \frac{M^2}{4}\chi_0^2, \qquad \underline{V_\chi(0) = M^2 d\,\chi,} \qquad V_{\chi,\chi}(0) = -M^2$$

At the top of the Higgs potential

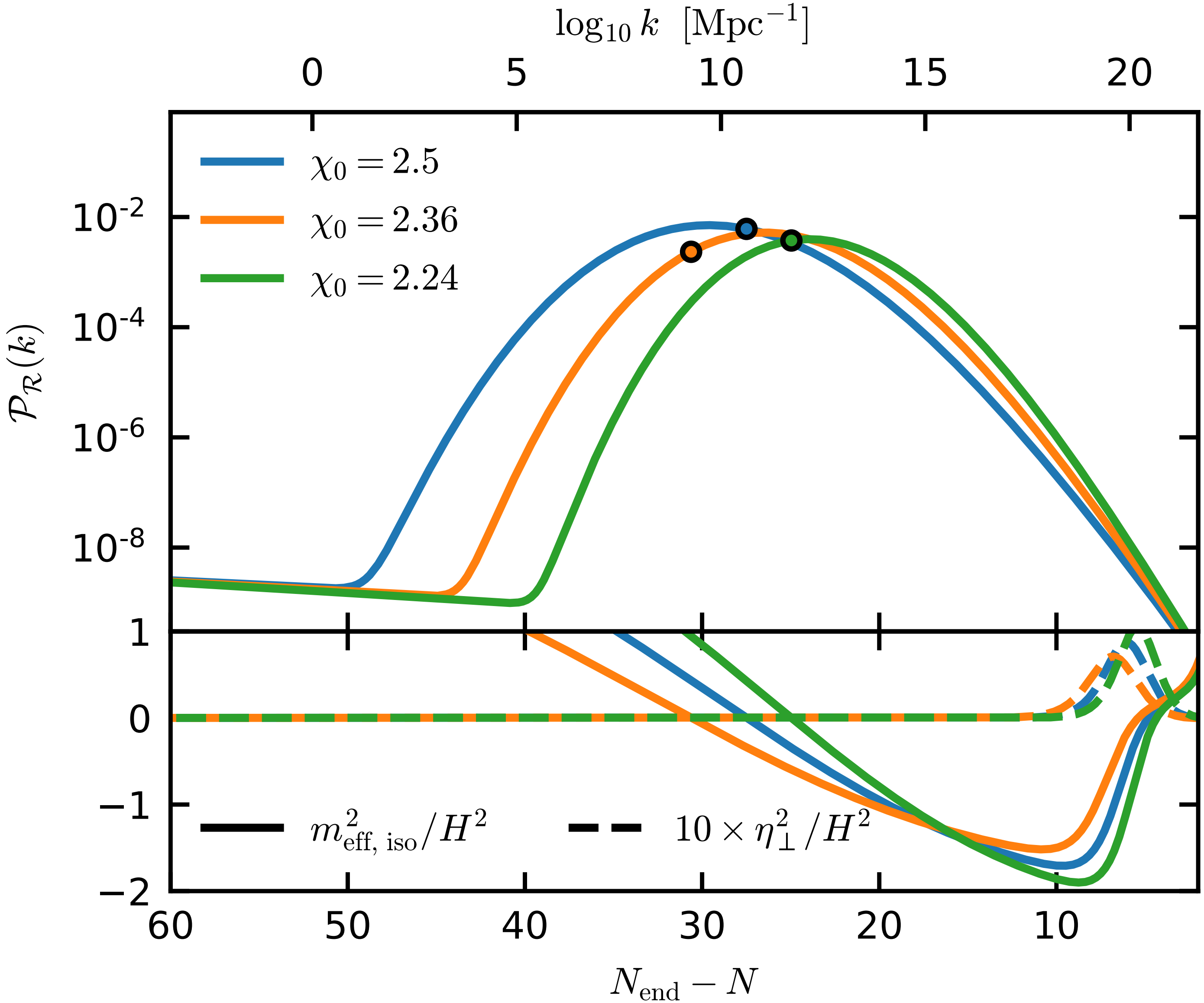
$$A_s = \frac{V^3}{12\pi^2 V_\chi^2} < 1 \quad \text{if} \quad d \gtrsim \frac{M\chi_0^3}{2^4\sqrt{3}\pi}$$

The linear term allows to avoid eternal hybrid inflation and superheavy topological defects. By a proper choice of d one can dial perturbations with a peak of any desirable height with the wavelength controlled by d, χ_0, and other parameters.

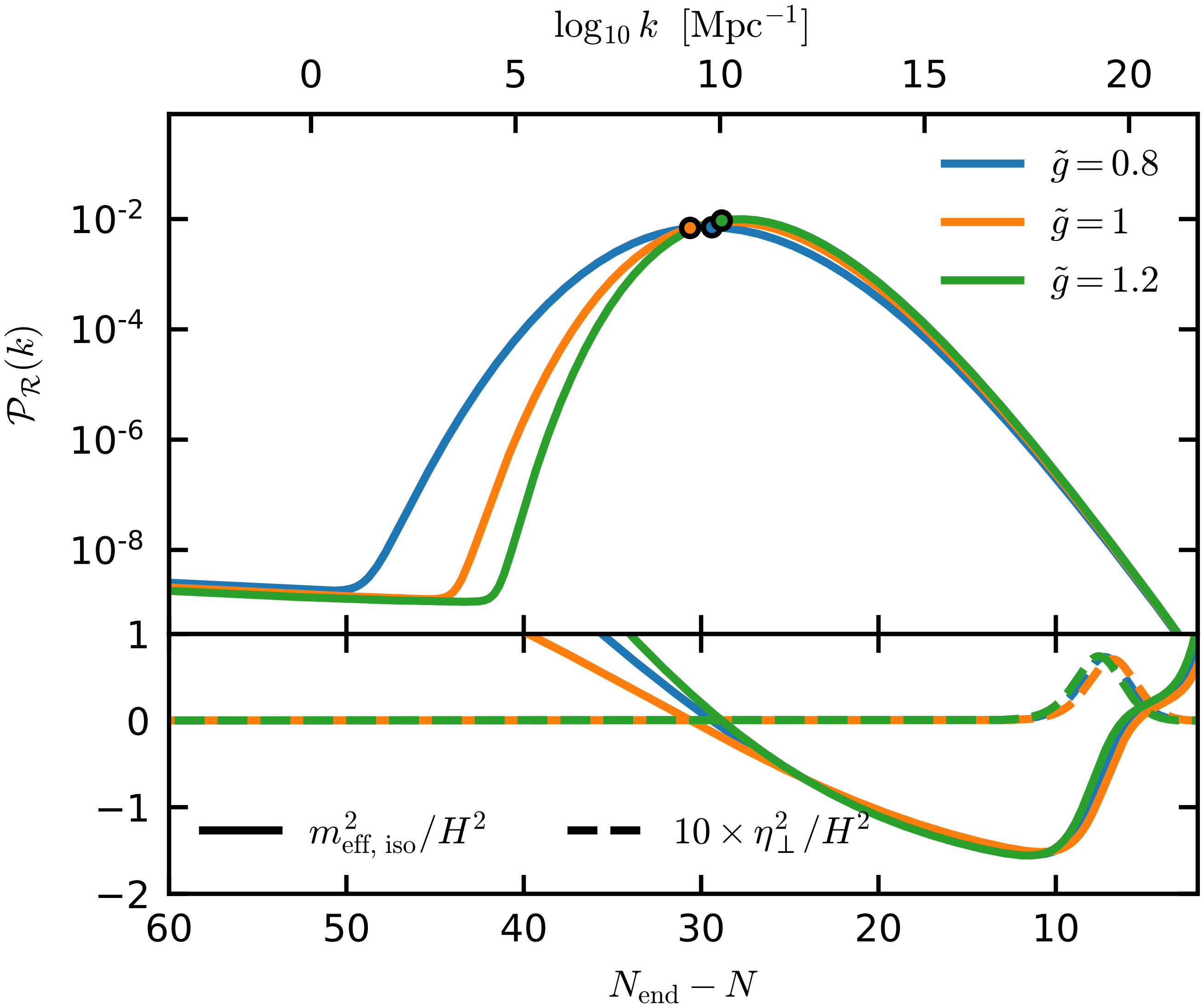
A detailed theory of perturbations generated during the two-field evolution is quite complicated, but the qualitative conclusion obtained above remains valid.



The amplitude of perturbations and position of the peak as a function of the linear term ~ d.



Change of the position of the peak due to the change of χ_0

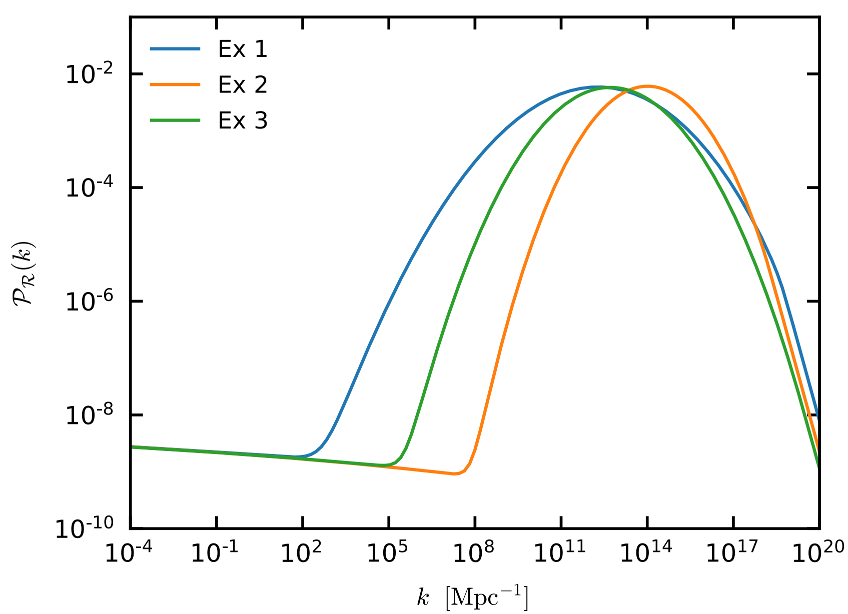


Changing the width of the peak by changing the coupling constant g of the interaction between the two fields χ and ϕ

Similar results for the polynomial (KKLTI) hybrid attractors

$$V_{\rm poly}(\chi,\varphi) = M^2\left[\frac{(\chi^2-\chi_0^2)^2}{4\chi_0^2} + \underline{\frac{\tilde{m}^2}{2}\frac{\alpha^2\varphi^2}{\varphi^2+\alpha^2}} + \frac{\tilde{g}^2}{2}\varphi^2\chi^2 + d\chi\right]$$

	d	$\tilde{m}\alpha$	$\tilde{g}$	ΔN	$n_{s,\rm KKLTI}$	$n_{s,\rm num}$
Example 1	-2×10^{-7}	0.7	2	9.43	0.9671	0.9670
Example 2	-10^{-7}	0.3	6	11.00	0.9659	0.9652
Example 3	-6×10^{-8}	0.05	8	8.91	0.9674	0.9662



The models discussed above have been formulated in supergravity context. They allow many interesting generalizations, not only for inflation but also for the theory of dark energy.

It is possible to avoid adding the linear term d χ if, for example, instead of a single field χ one considers a theory of several fields χ_i and controls the amplitude of perturbations by the symmetry breaking parameter $\chi_0 < 1$.

Our main goal here was to show that one can construct hybrid inflation models which fit Planck data and can produce perturbations with a very high peak, which may either result in PBH production, or to the stochastic gravitational wave background.